JOINTS FOR TRIANGULAR TRUSSES USING RECTANGULAR TUBES.

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INTRODUCTION

Triangular trusses comprising a single tension chord and two compression chords have been used as exposed roof structures in a number of recent applications. These trusses have considerable visual appeal, especially when fabricated from hollow structural sections (HSS) with a Warren arrangement of members in the two web planes, and when no joint reinforcement is necessary. Their inherent lateral and torsional stiffness can result in complete absence of lateral bracing, further enhancing their appearance and facilitating erection. While triangular truss systems may not exhibit economic advantages over planar systems, the attractive appearance of these trusses is frequently a determining factor in their choice. Triangular truss systems are aesthetically similar to space truss systems and in many applications they can be fabricated and erected at lower cost.

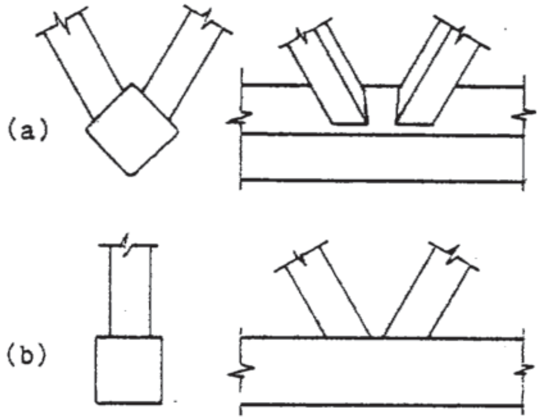


Fig.1. Planar and Triangular Truss Tension Chord Joints.

Triangular trusses fabricated from HSS members will require joint geometries quite different from those in planar HSS trusses. Tension chords may have web or bracing members attached to two adjacent walls, Fig.1(a), whereas in a planar truss only one wall is normally loaded, as shown in Fig.1(b). In the following, test programs are briefly described in which the behaviour of such tension chord joints was investigated. The mechanics of joint deformations are discussed in relation to yield line theory, and this theory is shown to give a good prediction of the joint behaviour.

Because gap joints cause more severe loading on the chord than overlap joints, and because of the fabrication advantages of the former, attention is restricted herein to gap joints.

TEST PROGRAMS

Two experimental programs were carried out in order to investigate the behaviour of the triangular truss tension chord double-K (DK) joints. In one program sixteen double-T (DT) joints were tested modeling in a simplified form the double-K joints. In the other program, seven triangular truss segments were tested to simulate the conditions the double-K joint would experience as part of a complete truss. These programs are briefly described below.

Simplified Joint Tests

Tests were carried out on isolated double-T joints as shown schematically in Fig.2. This simplified joint has two compression web members connected perpendicularly to the chord axis, and no tension web members. It simulates the main features of a double-K joint for which the primary deformations are associated with the compression web members. Three planar T joints, each one having one compression web member, were included in order to relate the behaviour of the DT joints to the planar T joint in which only one wall of the chord is loaded. All chords were square HSS with size of 127.0, 154.0 or 177.8 mm (5, 6 or 7 in.) and thickness of either 4.78 or 7.95 mm (3/16 or 5/16 in.). The principal variables were the web member width to chord width ratio, β , which varied from 0.2 to 0.6, the chord wall width to thickness ratio, b_0/t_0, equal to 26.6 for most specimens, and the angle between web members, α , which varied from 30° to 90°.

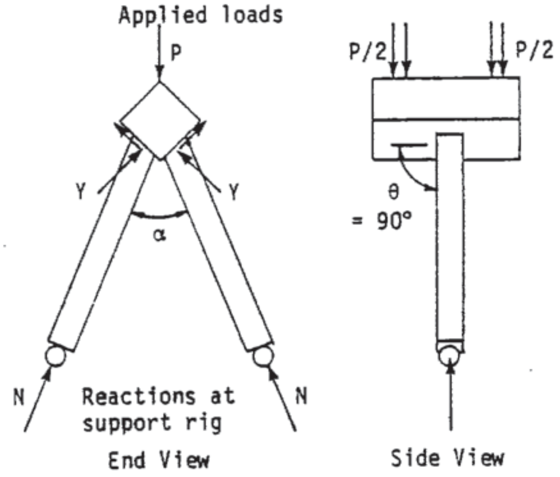


Fig.2. Simplified Joint Test Arrangement.

The total load, P, applied on the chord was recorded together with joint deformations measured parallel to the axes of the web members. These deformations resulted from the flexure of the chord wall face due to the punching action of the web members.

Truss Segment Tests

Tests were carried out on short lengths of triangular truss, each representing the end panel of a simply supported Warren type truss. A truss segment of this type is shown schematically in Fig.3 in the inverted position in which the specimens were tested. This truss panel was designed primarily to investigate the double-K joint behaviour. All tension chords were HSS - 127.0 x 127.0 (5 x 5 in.) sections with wall thickness of either 4.78 mm or 7.95 mm (3/16 or 5/16 in.). The principal variables were β , with values of 0.4 and 0.6, b_0/t_0, with values of 16 and 25.6, and α , with values of 60° and 90°.

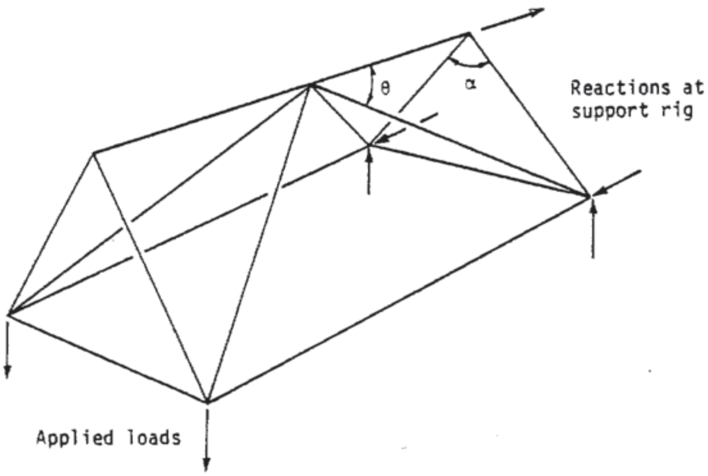


Fig.3. Truss Segment Test Arrangement.

The truss segments were cantilevered from a massive rig which was bolted to the laboratory reaction floor. Loads applied vertically at the free end of the truss were recorded. Joint deformations were measured parallel to the axes of both the compression and tension web members. Full details of the test specimens, instrumentation and loading are given in Ref. (1).

Test Observations:

The observed failure mode consisted primarily in the punching in of the chord connected walls either with or without crippling failure of the chord unconnected walls .

Loads were derived from the test results in terms of the normal load, Y, on the chord wall arising from the web member attached to it (see Figs. 2 and 16). The joint load-deformation curves, as shown typically in Fig.4, were used to find the following test loads:

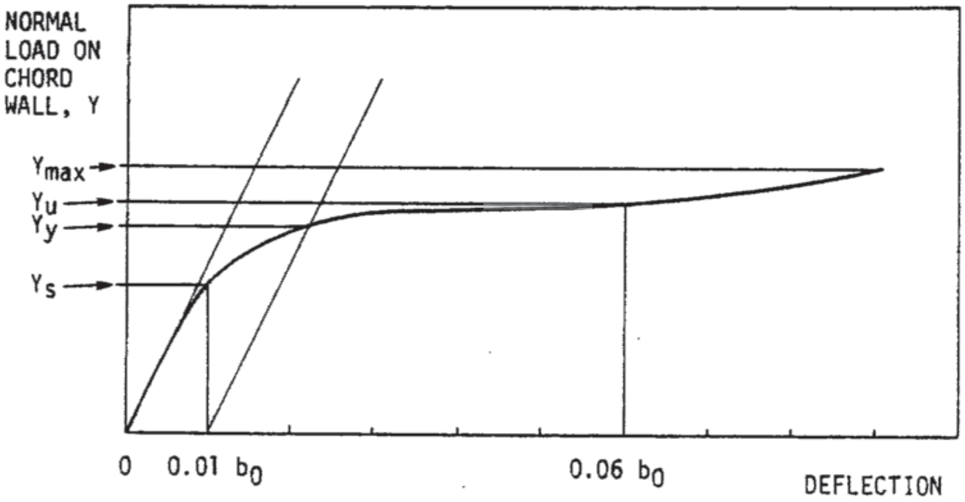


Fig.4. Typical Joint Deformations.

(a) A failure load Y_u, taken as the load corresponding to a chord wall deflection of 0.06 b_0.

(b) A yield load Y_y, corresponding to the region of the load-deflection curve where deflections started to increase rapidly. This load is defined by the offset shown in Fig.4.

(c) A service load Y_s, corresponding to a chord wall deflection of 0.01 b_0.

Further detailed observations are given in Refs. (2) and (3).

In Fig.5 the deformed chord cross-section at the joint of a truss specimen is shown. Rotation of the chord walls can be clearly seen. This arises primarily because geometrical considerations lead to off-center placement of the web members in order to minimize the offset between web member and chord axes.

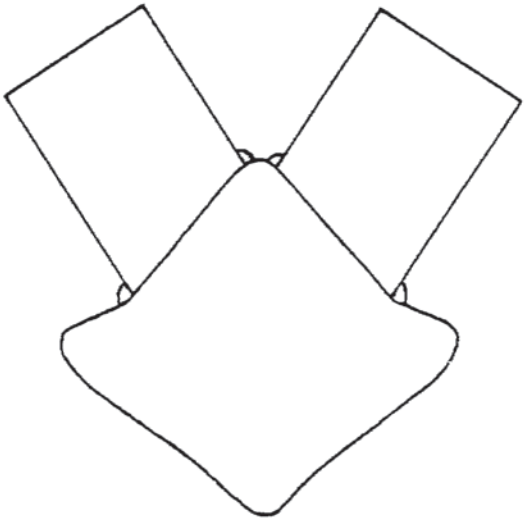


Fig.5. Cross-Section Distortion - DK Truss No.4.

YIELD LINE ANALYSIS

Test results are compared with the yield line method of analysis. This approach has been justified both on the basis that it corresponds to a reasonable deflection magnitude at service load level(4,5), and because it predicts a "yield load" which is in reasonable agreement with test loads at which the growth of joint deformations accelerates(6,7). The yield line method, and its application to the test specimens is summarized below. This summary gives the equation for the yield load of a T joint, and various modifications to that equation to account for the inclination of the web members to the chord wall, and for the off-center position of the web member for double-T joints. Results are also given for K and double-K joints.

Yield Line Theory

For the yield line model and loading illustrated in Fig.6 the yield line method gives the perpendicular load on the chord wall at yielding failure of the wall as (6)

$$Y_y = 8m_p \left[\frac{\eta}{(1-\beta)} + \frac{2}{\sqrt{1-\beta}} \right] \qquad \ldots (1)$$

in which $m_p = 0.25t_0{}^2F_{y0}$, the plastic moment per unit width of the chord wall, b_0 is the chord wall width, $\eta = h_1/b_0$ and β is the width ratio b_1/b_0. The web member outside dimensions are $h_1 x b_1$ with the width b_1 measured perpendicular to the chord axis.

To account for the weld size and the curvature at the corners of the chord, the nominal width ratio is adjusted as follows (see for example Ref.(7)): $\beta' = b_1'/b_0'$, where $b_1'$and b_0' are defined in Fig.7. Similarly $\eta' = h_1'/b_0'$

Application to Double-T Test Specimens

Eq.(1) can be applied to the double-T joint arrangement by adjustment of the width b_1' to account for the slightly wider contact width in those cases where $\alpha \neq 90^\circ$ (see Fig.8). The yield load normal to a chord wall then becomes

$$Y_y = 8m_p \left[\frac{\eta}{1-\beta\sec(45^\circ-0.5\alpha)} + 2\sqrt{\frac{1}{1-\beta\sec(45^\circ-0.5\alpha)}} \right] \qquad \ldots (2)$$

For many of the double-T specimens, the web members were not centered on the middle of the chord wall in order to minimize eccentricity. The essential effects of this can be determined by analysing the yield line model shown in Fig.9. Assuming a normal load on the chord wall, and no rotation of the web member relative to the chord, the yield load becomes

$$Y_y = 8m_p \left[\frac{\eta}{4\xi(1-\frac{\xi}{1-\beta})} + \frac{1}{\sqrt{\xi}\sqrt{1-\frac{\xi}{1-\beta}}} \right] \qquad \ldots (3)$$

in which $\xi = a/b_0$. The dimensions a and b_0 can again take account of the corner curvature and the weld size, and β should be replaced by $\beta \sec(45^\circ - 0.5\alpha)$ for cases where the angle between web planes is less than 90°.

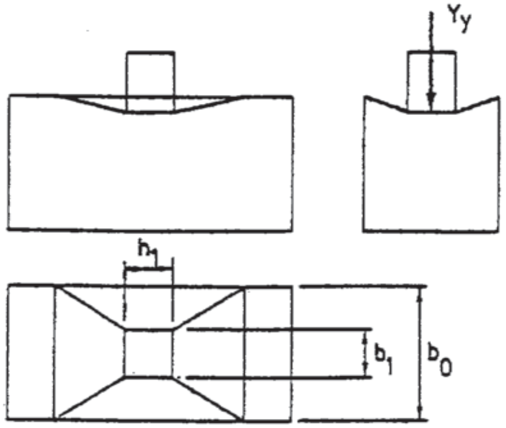


Fig 6. Simple Yield Line Model

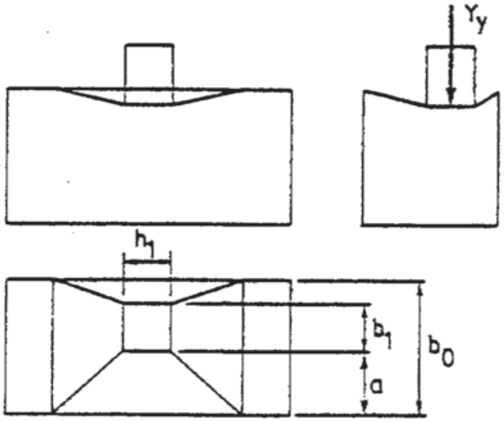


Fig 9. Simple Yield Line Model for Off-Centre Web Members

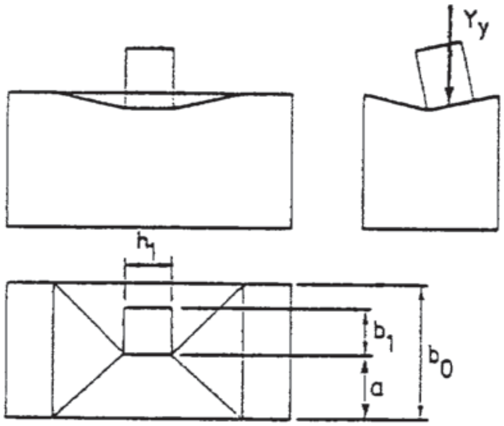
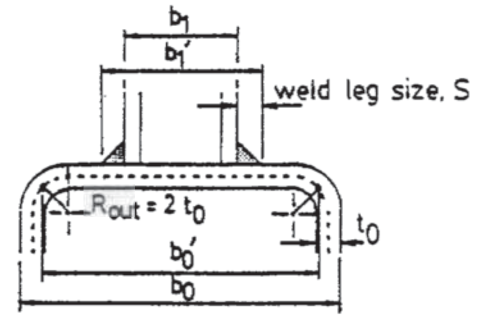


Fig 10. Alternative Yield Line Model for Severely Off-Centre Web Members



$b_1' = b_1 + 2S$, (similarly $h_1' = h_1 + 2S$)

$b_0' = b_0 - (4 - 1.5\sqrt{2})t_0 = b_0 - 1.88\,t_0$

t_0: wall thickness of chord

Fig 7. Modification for Corner Curvature and Welds

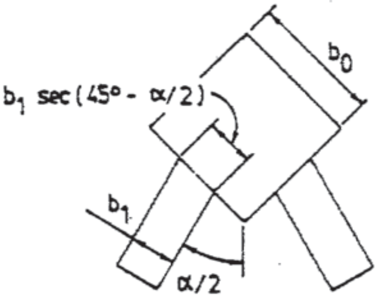


Fig 8. Modification for Test Models when $\alpha \neq 90°$

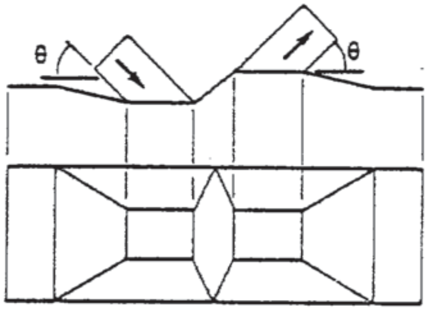


Fig 11. Yield Line Model for Planar Truss Joint

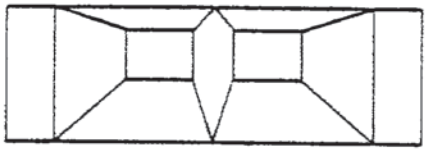


Fig 12. Yield Line Model for Off-Centre Truss Joint

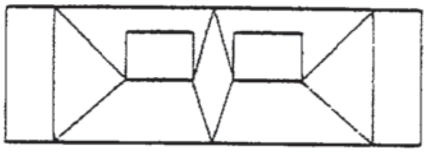


Fig 13. Alternative Yield Line Model for Severely Off-Centre Truss Joint

When the web member is significantly off-center, it becomes more likely that a different mechanism (from that of Fig.9) involving rotation of the web member in a plane normal to the chord axis takes place. A simple representative mechanism is shown in Fig.10. Assuming that no work is done in the rotation (i.e. that the bending moment in the web member is zero), the yield load normal to the chord wall can be written

$$Y_y = \frac{8m_p}{[2(1-\xi) - \beta]} \left[\frac{\eta}{2\xi} + \frac{2\sqrt{1-\xi}}{\sqrt{\xi}} \right] \qquad \ldots (4)$$

The appropriate value of the theoretical yield load is the lower of the two upper bound solutions Eqs. (3) and (4).

Application to K and Double-K Joints

K Joints: Yield line analysis in its simple form has been applied to K and N planar truss joints by Davies and Roper (8) and Mouty (4). For trusses with equal sized tension and compression web members with equal inclinations θ to the chord axis, the yield line mechanism shown in Fig.11 gives the following yield load normal to the chord wall

$$Y_y = 8m_p \left[\frac{\eta \operatorname{cosec}\theta}{1-\beta} + \frac{1}{\sqrt{1-\beta}} + \frac{1}{4\gamma} + \frac{\gamma}{2(1-\beta)} \right] \qquad \ldots (5)$$

where $\gamma = g/b_0$ in which g is the gap dimension, and $\eta = h_1/b_0$, etc.

Double-K Joints: For cases in which the web members are attached centrally on the chord walls of a triangular truss, Eq.(5) must be modified to account for the greater width of attachment of the web member to the chord. This can be achieved by replacing β in these equations by $\beta \sec(45^\circ - 0.5\,\alpha)$.

When the web members are attached off-center, the yield line model illustrated in Fig.12 leads to the following yield load normal to the chord wall

$$Y_y = m_p \left[\frac{2\eta + \gamma}{\xi\,(1-\frac{\xi}{1-\beta})} + \frac{2}{\gamma} + \frac{4}{\sqrt{\xi\,(1-\frac{\xi}{1-\beta})}} \right] \qquad \ldots (6)$$

Again, $\xi = a/b_0$ and the dimensions can again be interpreted from Fig.7 to take into account the corner curvature and weld material. To incorporate the effects of the inclination of the web member to the chord axis, β is adjusted as before to $\beta \sec(45^\circ - 0.5\,\alpha)$ and in addition η is replaced by $\eta \operatorname{cosec}\theta$.

As for the double-T specimens, another mechanism corresponding to web members attached near the corner of the chord may give a lower yield load than Eq.(6). This mechanism is illustrated in Fig.13, and gives the following yield load:

$$Y_y = \frac{4m_p}{[2(1-\xi) - \beta]} \left[\frac{\eta}{\xi} + \frac{2\sqrt{1-\xi}}{\sqrt{\xi}} + \frac{(1-\xi)}{\gamma} + \frac{\gamma}{2\xi} \right] \qquad \ldots (7)$$

DISCUSSION

Comparison between Test and Theory

Test results, expressed as the normal loads on the chord walls Y_u and Y_y are compared with the yield line theory results on Fig.14. This comparison includes the full range of values of the parameters tested. While there is some scatter, it can be seen that there is a considerable measure of agreement. For the sixteen simplified joint tests, Fig.14(a), the mean value of the ratio of test and theoretical yield loads Y_y is 1.043 with coefficient of variation 11.8%. For the ultimate loads (corresponding to 0.06 b_0 deflection) the corresponding figures are 1.184 and 8.2%. For the four double-K trusses, Fig.14(b), the ratio of experimental yield load to theoretical value varies from 1.03 to 1.28.

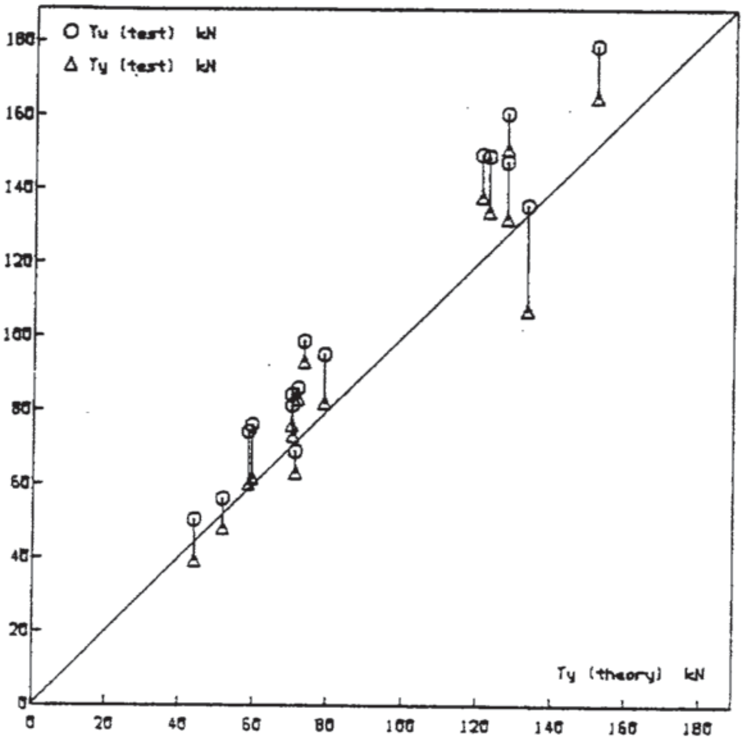


Fig.14(a). Test Values of the Normal Loads on the Chord Walls Y_u and Y_y vs. Yield Line Theory Results.
Double-T Joint Tests.

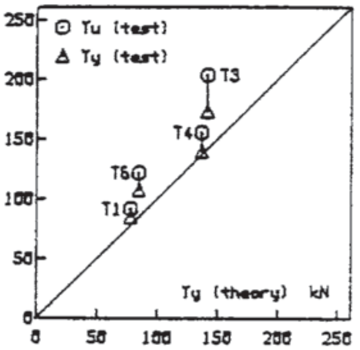


Fig.14(b). Test Values of the Normal Loads on the Chord Walls Y_u and Y_y vs. Yield Line Theory Results.
Double-K Truss Segment Tests.

Note: Y is the normal load on the chord wall arising from the web member attached to it and the subscripts 'u' and 'y' are defined in Fig.4.

Examination of the test to theoretical yield and ultimate load ratios showed no discernable trends over the tested ranges of the parameters α, β and b_0/t_0 suggesting that the yield line method adequately incorporates the parameters α and β, and that the ratio b_0/t_0 does not have a significant influence if it lies within the range 20 to 37.

Joint Deformations

A joint deformation of 1% of the chord width b_0 has been suggested as an appropriate deflection limit at service load level for planar HSS truss joints (4,5). The test loads at which this deformation was recorded are compared in Fig.15 with the yield load predicted by the yield line theory. For the double-T joints the mean value of the ratio of these measured loads to the theoretical yield line value is 0.625, with standard deviation 0.163. A line corresponding to this mean is shown on Fig.15(a).

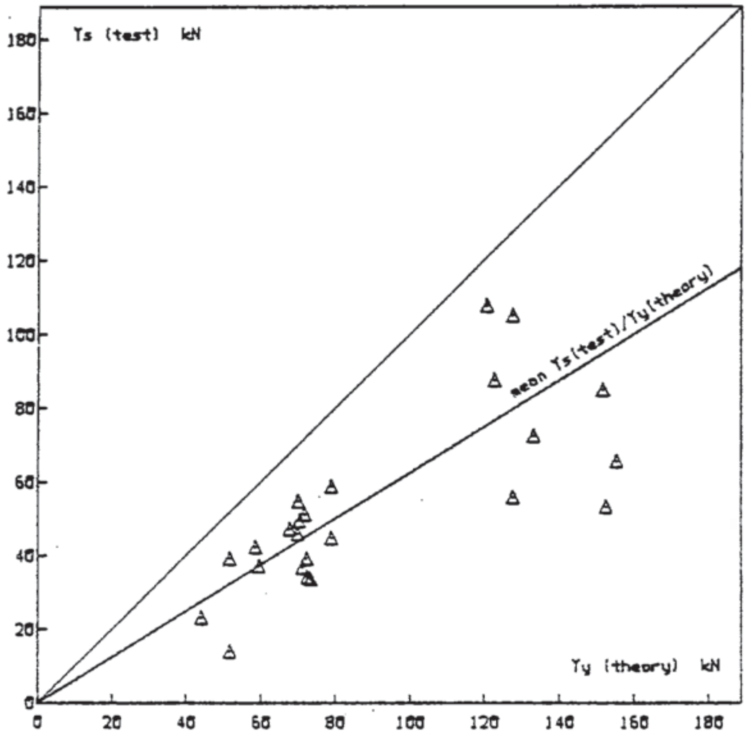


Fig.15(a). Test Service Loads (at a Joint Deformation of 1% b_0) vs. Yield Line Theory Results. Double-T Joint Tests.

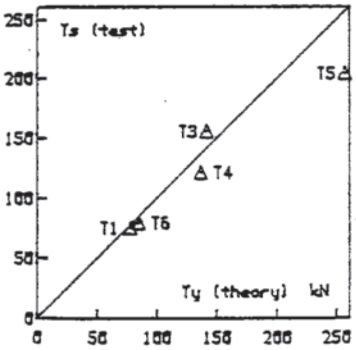


Fig.15(b). Test Service Loads (at a Joint Deformation of 1% b_0) vs. Yield Line Theory Results. Double-K Truss Segment Tests.

Note: Y is the normal load on the chord wall arising from the web member attached to it and the subscript 's' is defined in Fig.4.

For the double-K joints the load corresponding to deflections of 0.01 b_0 are compared with the yield line theory on Fig.15(b). Close correspondence exists, and it can be seen that the spread between this load and that corresponding to 0.06 b_0 deflection, shown in Fig.14(b), is much smaller than for the double-T specimens. At working load levels therefore the deflections will be considerably lower than 0.01 b_0.

Effect of Loading on Two Adjacent Chord Walls

The effect, in triangular trusses, of loading on two chord walls, compared with the loading on one chord wall which occurs in planar trusses can be evaluated by comparing results for planar T joints to double-T joints with 90° angles between web planes.

Two pairs of the relevant test specimens can be compared, for which the ratio of failure loads is 0.99 and 0.95. These results, for which β was 0.2 and 0.4 and $b_0/t_0 = 26.6$, suggest that there is little interactive effect produced by identical loading on an adjacent wall of the chord. Because of this, for cases where the angle between web planes is less than 90° leading to an increase in the effective value of β, and also when web members are attached to the chord wall off-center, the strength of a triangular truss tension chord wall will be greater than that of a planar truss chord with the same size members.

DESIGN EXAMPLE

The following example of the application of the above analysis is based on a load and resistance factor design approach, as used in Canada. Verification of the ultimate limit state (maximum strength) and the serviceability limit state (e.g. deflections under service loads) is required. The ultimate limit state condition of a structural element is satisfactory if the factored resistance of the element is greater than the factored loads acting on it:

Factored Resistance $\geqslant$ Factored Loads
i.e. ϕ (Resistance) $\geqslant$ α (Loads)

where ϕ is a resistance factor, usually taken as 0.9 in Canada, and α represents load factors, usually of value between 1.05 and 1.5.(9)

In the following, the ultimate state condition of a triangular truss joint is checked using the yield line equations derived earlier to check the bending resistance of the chord connected walls.

The joint considered is shown in Fig. 16. The member axial forces, as determined by an analysis of the truss assuming pinned joints, and the preliminary selection of member sizes is also shown this Figure.

Factored Loads:

$Tf_1 = 423.5$ kN (95.2 Kips)
$Y_{f1} = N_1 \sin\theta_1 \cos(45^{\circ} - \alpha/2) = 207\text{kN} \sin(59.23^{\circ}) \cos(15^{\circ})$
$= 171.8$ kN (38.5 Kips)
$Tf_2 = 0$
$V = 308$ kN (69.2 Kips)
$N_1 = 207.0$ kN (46.5 Kips) (compression)
$N_2 = 207.0$ KN (46.5 Kips) (tension)

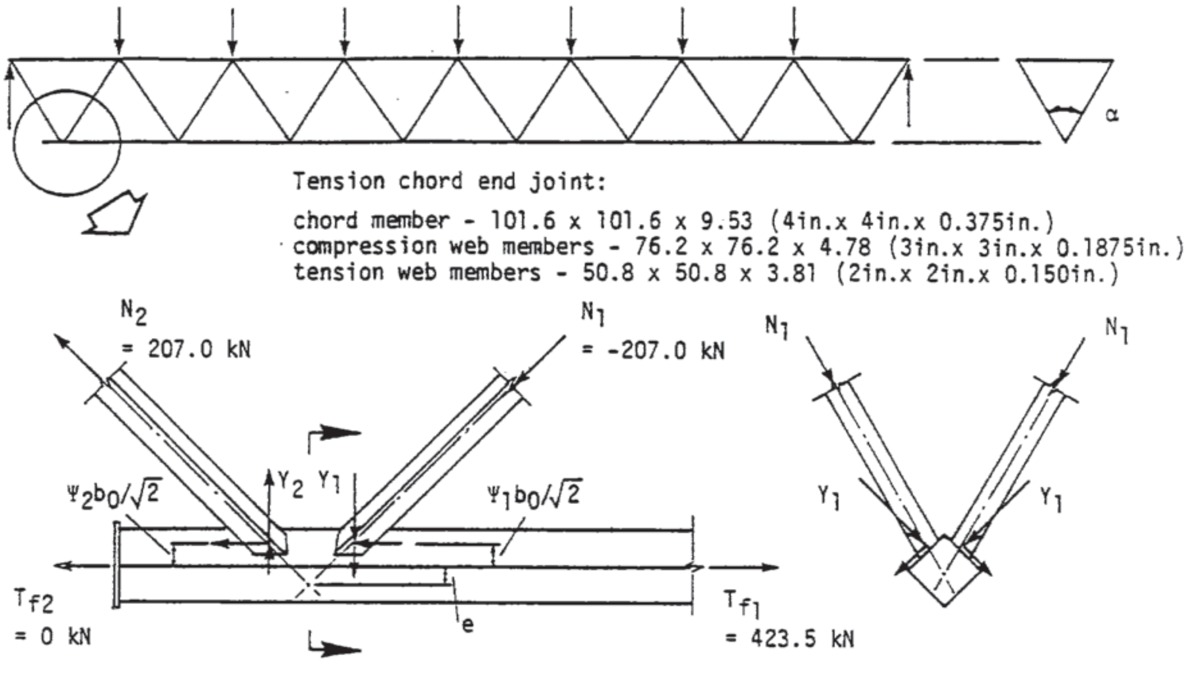
Geometrical Data:

$b_0 = 101.6$ mm (4 in.)
$t_0 = 9.53$ mm (0.375 in.)
$b_1 = h_1 = 76.2$ mm (3 in.)
$b_2 = h_2 = 50.8$ mm (2 in.)
$\theta_1 = \theta_2 = \theta = 59.23^{\circ}$, $\alpha = 60^{\circ}$
$a_1 = 11.36$ mm (0.447 in.)
$a_2 = 37.7$ mm (1.484 in.)
$g = 20$ mm (0.787 in.)
$F_{y0} = 350$ Mpa (50 Ksi) (yield stress of steel)

The plastic moment per unit width of the chord wall is

$m_p = 0.25\, t_0^2\, F_{y0} = 0.25\, (9.53 \text{ mm})^2\, 350 \text{ N/mm}^2$
$= \underline{7.947 \text{ kN.m/m}}$ (1.785 Kip-in./in.)

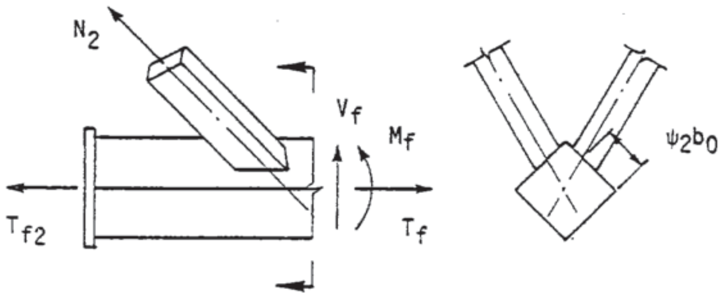
The β ratio is taken, conservatively, as the average width of the tension and compression web members, as commonly recommended in planar truss joint design. For simplicity and also because it is conservative, the weld size and the curvature at the corners of the chord are not considered.



Tension chord end joint:

chord member - 101.6 x 101.6 x 9.53 (4in.x 4in.x 0.375in.)
compression web members - 76.2 x 76.2 x 4.78 (3in.x 3in.x 0.1875in.)
tension web members - 50.8 x 50.8 x 3.81 (2in.x 2in.x 0.150in.)

Forces on tension chord



Forces in the gap

Fig.16. Design Example - Tension Chord End Joint.

$$\beta \sec(45^\circ - 0.5\alpha) = \frac{b_1 + b_2}{2b_0} \sec(45^\circ - 0.5\alpha)$$

$$= \frac{76.2 \text{ mm} + 50.8 \text{ mm}}{2(101.6 \text{ mm})} \frac{1}{\cos(45^\circ - 0.5(60^\circ))} = \underline{0.647}$$

$$\eta \operatorname{cosec} \theta = \frac{h_1 + h_2}{2b_0} \operatorname{cosec} \theta = \frac{76.2 \text{ mm} + 50.8 \text{ mm}}{2(101.6 \text{ mm})} \frac{1}{\sin(59.23^\circ)} = \underline{0.727}$$

$$\xi = \frac{a_1}{b_0} = \frac{11.36 \text{ mm}}{101.6 \text{ mm}} = \underline{0.112}$$

$$\gamma = \frac{g}{b_0} = \frac{20 \text{ mm}}{101.6 \text{ mm}} = \underline{0.197}$$

Using Equation (6):

$$Y_y = m_p \left[\frac{2\,\eta + \gamma}{\xi(1 - \xi\,/(1-\beta))} + \frac{2}{\gamma} + \frac{4}{\sqrt{\xi(1 - \xi\,/(1-\beta))}} \right] =$$

$$= 7.947\,\frac{\text{kN.m}}{\text{m}} \left[\frac{2(0.727) + 0.197}{0.112\,(\,1 - 0.112/(1-0.647)\,)} + \frac{2}{0.197} + \right.$$

$$\left. + \frac{4}{\sqrt{0.112\,(\,1 - 0.112/(1-0.647)\,)}} \right] = \underline{367.3\ \text{kN}}\ (82.5\ \text{Kips})$$

Using Equation (7):

$$Y_y = \frac{4\,m_p}{2(1-\xi) - \beta} \left[\frac{\eta}{\xi} + \frac{2\sqrt{1-\xi}}{\sqrt{\xi}} + \frac{1-\xi}{\gamma} + \frac{\gamma}{2\xi} \right] =$$

$$= \frac{4(7.947\ \text{kN.m/m})}{2(1-0.112) - 0.647} \left[\frac{0.727}{0.112} + \frac{2\sqrt{1-0.112}}{\sqrt{0.112}} + \frac{1-0.112}{0.197} + \frac{0.197}{2(0.112)} \right]$$

$$= \underline{493.1\ \text{kN}}\ (110.8\ \text{Kips})$$

Hence the correct value of Y_y is 367.3 kN (82.5 Kips) and finally

$$\phi\ (367.3\ \text{kN}) = 0.9\ (367.3\ \text{kN}) = 330.6\ \text{kN} \geqslant 171.8\ \text{kN} \quad \therefore \text{Satisfactory}$$

$$(74.3\ \text{Kips} \geqslant 38.6\ \text{Kips})$$

In the gap between tension and compression web members, the chord section is loaded by high tension and shear forces as well as bending moments. For the joint to be satisfactory the conditions in this region must also be checked. These and other aspects of triangular truss design are discussed elsewhere (2).

CONCLUSIONS

Seven double-K truss segments and sixteen double-T specimens were tested in an attempt to determine joint stiffnesses and strength, and at the same time to determine if any significant trends were evident due to variation of four major parameters. The yield line theory has been demonstrated to provide an excellent correlation with the test results and can be used as a basis for the development of design rules for joints to the single chord of a triangular truss.

ACKNOWLEDGEMENT

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