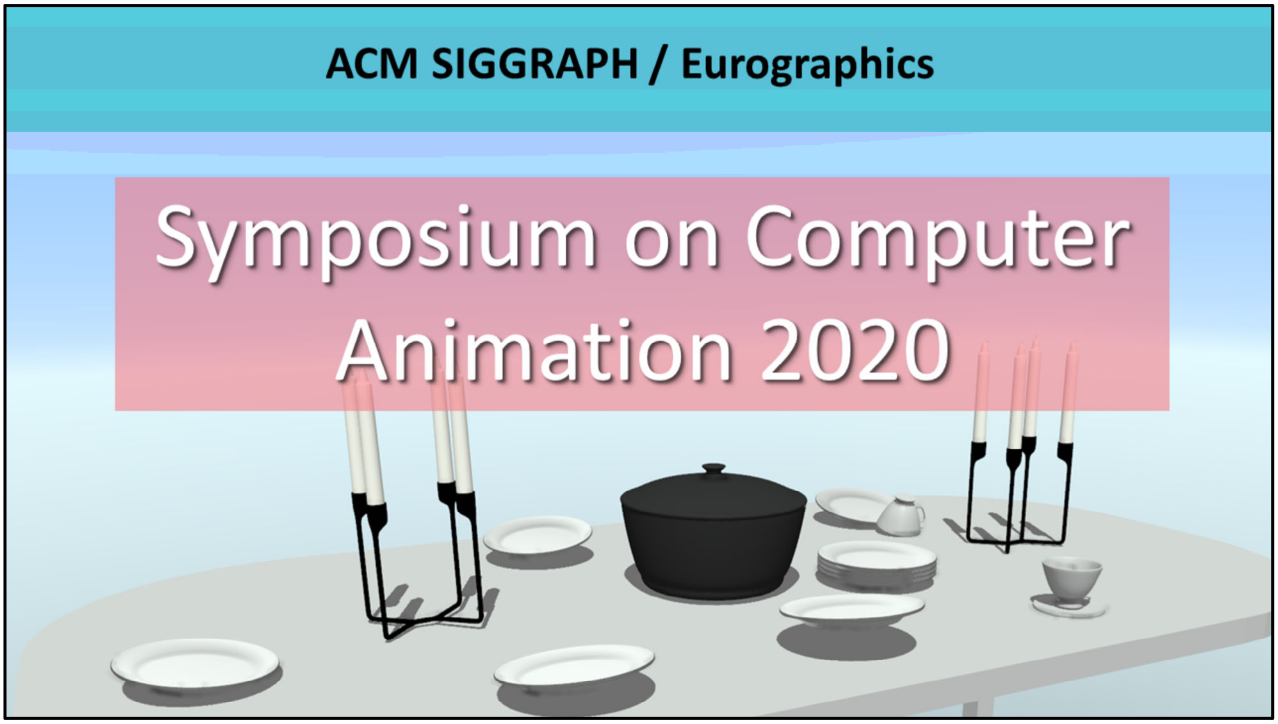


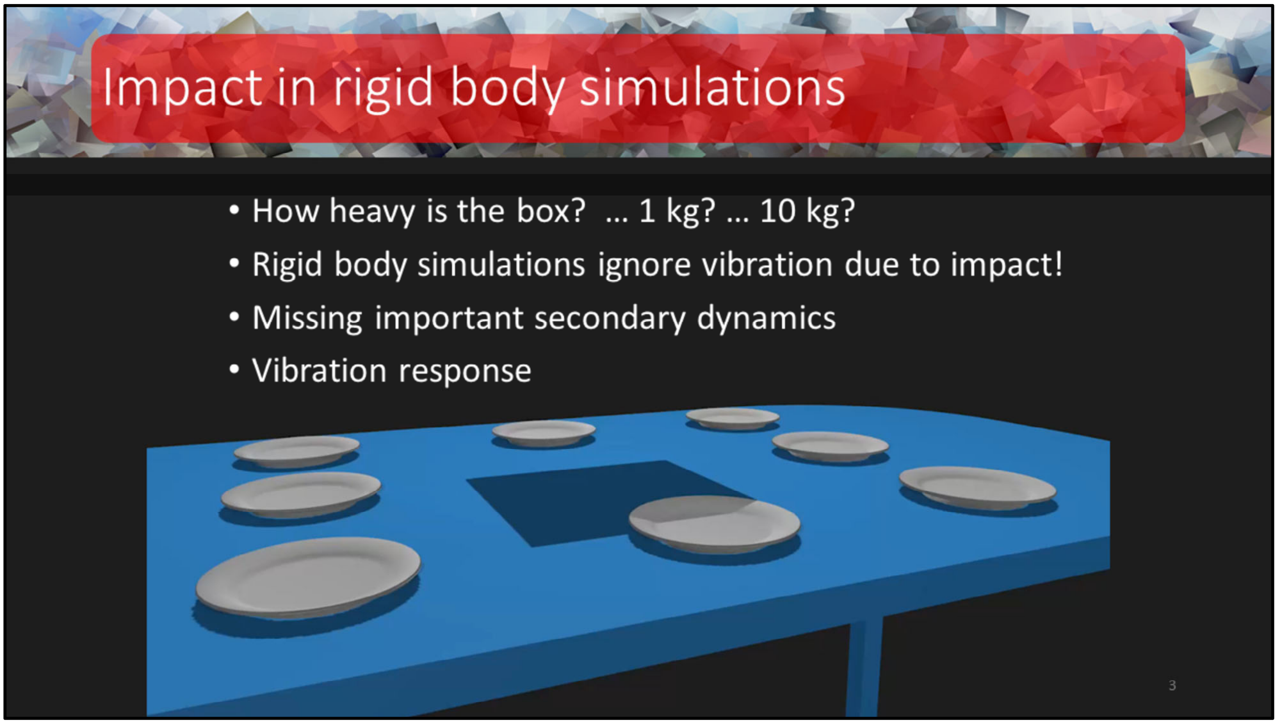

Hi everyone, my name is Sheldon Andrews, a professor at ETS in Montreal ….

# Distant Collision Response in Rigid Body Simulations

Eulalie Coevoet[1], Sheldon Andrews[2], Denali Relles[1], Paul G. Kry[1]

[1]McGill University
[2]École de technologie supérieure ÉTS

… and today I will be presenting about our work on Distant Collision Response for rigid body simulations.
This work was done in collaboration with my colleagues at McGill University– Eulalie Coevoet, Denali Relles, and Paul Kry.




So, let's begin by considering a rigid body simulation involving impact.

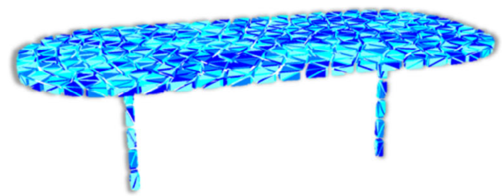
As we can see, the box in this scenario comes crashing down and collides with a table. Yet we are unable to discern important physical properties about the box. For instance, what is the mass of the box? Is it heavy or light?

Standard methods for rigid body simulations leave a lot to be desired in terms of the secondary dynamics that are exhibited in this type of example.
We therefore address this problem in our work. Specifically, we focus modeling the vibration response that arises due to an impact.

# Simulating vibrations

- Elastic simulations using FEM models, with mass $\mathbf{M}$, stiffness $\mathbf{K}$, and damping $\mathbf{D}$
- Small deformations $\mathbf{u} \in \mathbb{R}^{3n}$ due to loading $\mathbf{f} \in \mathbb{R}^{3n}$ follow the relationship:

$$\mathbf{M}\ddot{\mathbf{u}} = \mathbf{D}\dot{\mathbf{u}} + \mathbf{K}\mathbf{u} = \mathbf{f}$$



- Synthesizing vibrations requires small time steps ($h \ll 10^{-3}$)
- Computationally prohibitive for real-time



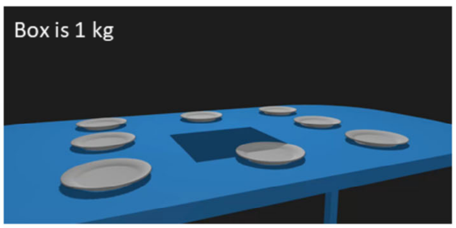
Typically, if we'd like to simulate vibrations due to impact, a detailed elastic model is required. Assuming small deformations, a linear elastic model involving the mass, stiffness, and damping of an FEM mesh could be used to synthesize vibrations.

However, to faithfully reproduce the high frequency vibrations that occur in stiff objects, very small time steps are necessary.

And even for linear elastic models, they are prohibitive for real-time applications such as video games and interactive VR training simulations, which we target in our work.

## Distant Collision Response (DCR)

- Compute the response to impact using a reduced elastic model
- Efficient computation by an IIR digital filter [James and Pai, 2002]
- Distribute the response by computing an impulse at each existing contact
- Efficient and suitable for real-time simulations with large time-steps







And this is where our contribution lies. We approximate the vibration response to collisions in rigid body simulations by using a pre-computed reduced elastic model based on modal FEM.

To render this process more efficient we compute the response using a digital IIR filter.

The response is computed as a displacement at FEM nodes that is then rendered as a impulse at existing contacts, causing disturbances to objects already resting on the surface of DCR bodies.

Our approach is efficient and is suitable for real-time applications with update rates on the order of 100 frames per second achievable. And as you can see, it really livens up rigid body simulations!


Our work is similar, but complementary, to work that has investigated enriching rigid body and kinematic simulations by storing auxiliary information in textures.

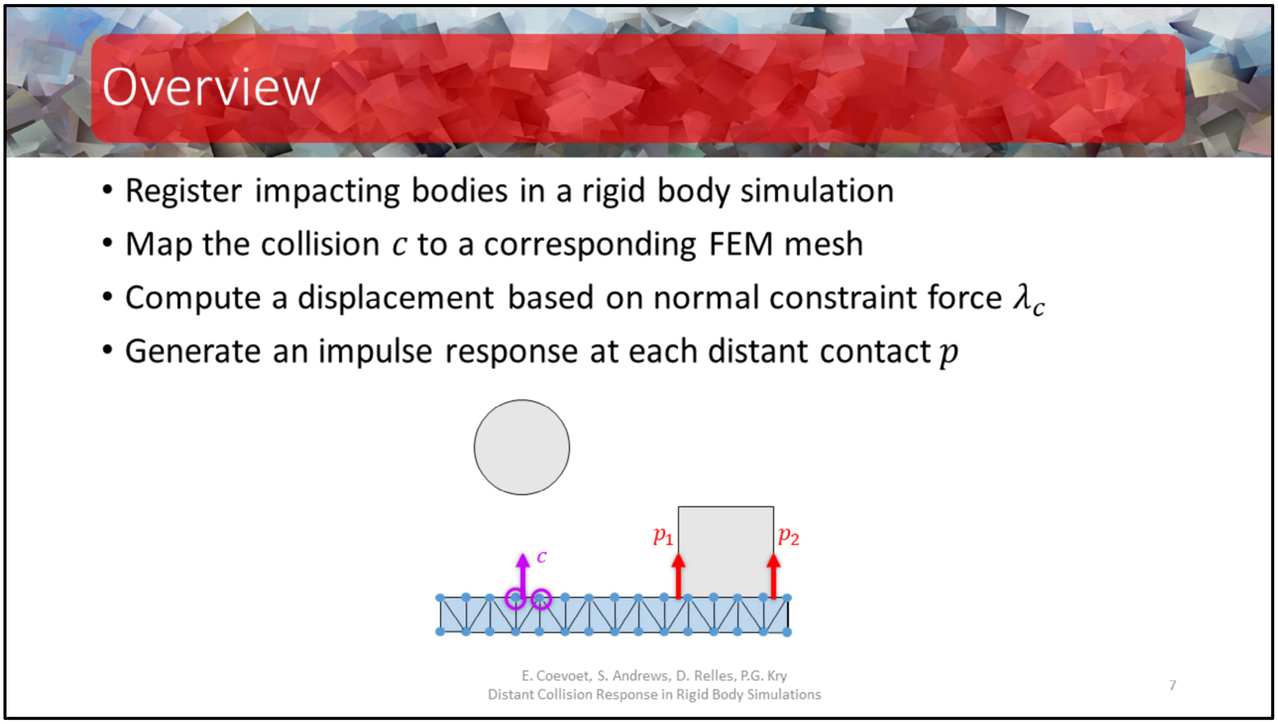
Notably, Bounce maps varied the coefficient of restitution across the surface of simulation bodies to produce more natural collision response based on the object geometry.

Our work also draw on concepts presented in the DyRT paper, which introduced modal vibrations that are excited by rigid body motion and digital IIR filter coefficients in textures assigned to the surface mesh. We similarly use a digital IIR filter to efficiently compute distant collision response.

Galoppo et al. also proposed elastic deformation textures to model a varying surface compliance for rigid bodies. Whereas our work use coefficients stored at nodal coordinates to compute a vibration response due to impact.

Furthermore, we are inspired by work on modal simulations for synthesizing sounds. For instance, the work by Zheng and James propose a method for modal sound simulations where all contacts are considered and boundary conditions are changed in order to produce high-quality contact sounds, albeit at a high computational cost. Conversely, our approach is fast to compute and we simplify the problem by considering each contact independently.

Other work in this genre has focused on generating sound for interactive simulations, such as FoleyAutomatic and the work by O'Brien and colleagues. Similar to our work, these approaches use a precomputed modal model, but with the goal of rendering plausible sounds due to collisions.



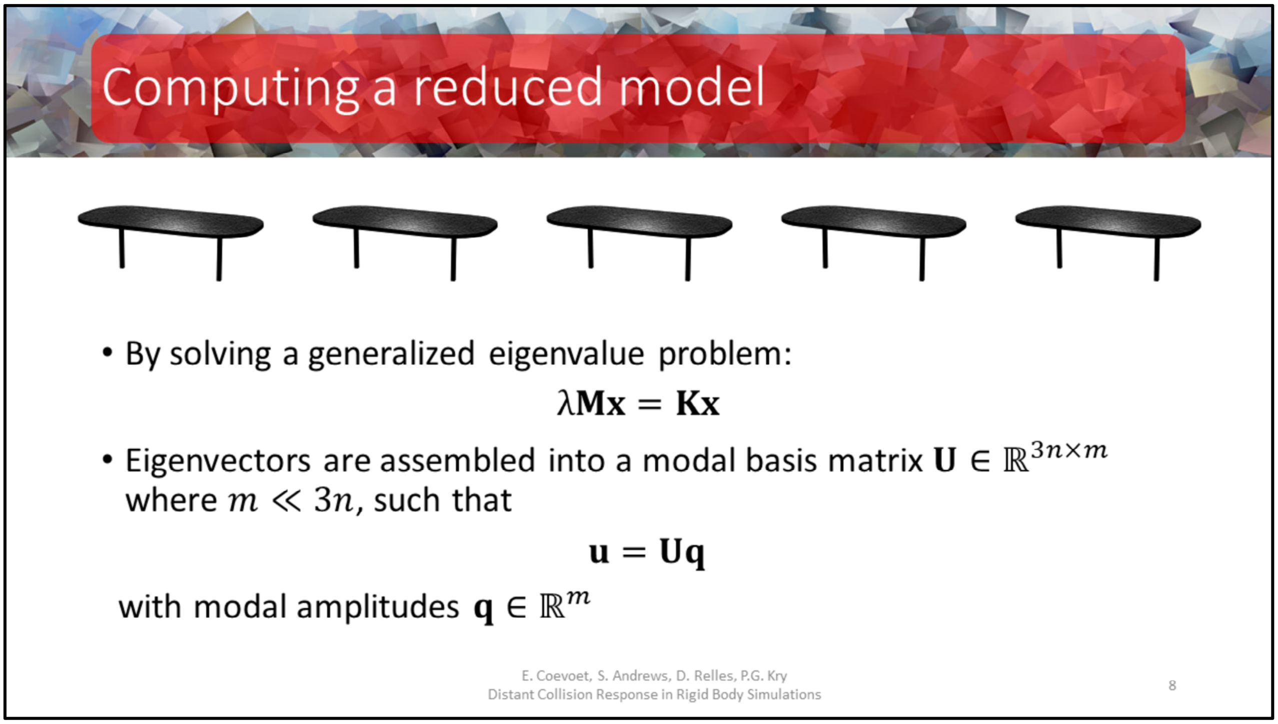

The first step in our method is to detect new collisions in the rigid body simulation that result in an impact.

Then, these impact collisions are mapped to an FEM mesh that corresponds to the collision proxy used in the rigid body simulation.

In our simulations, we use a collection of spheres, boxes, and convex meshes to render the rigid body simulation more efficient, but then compute corresponding points on the surface of an FEM mesh

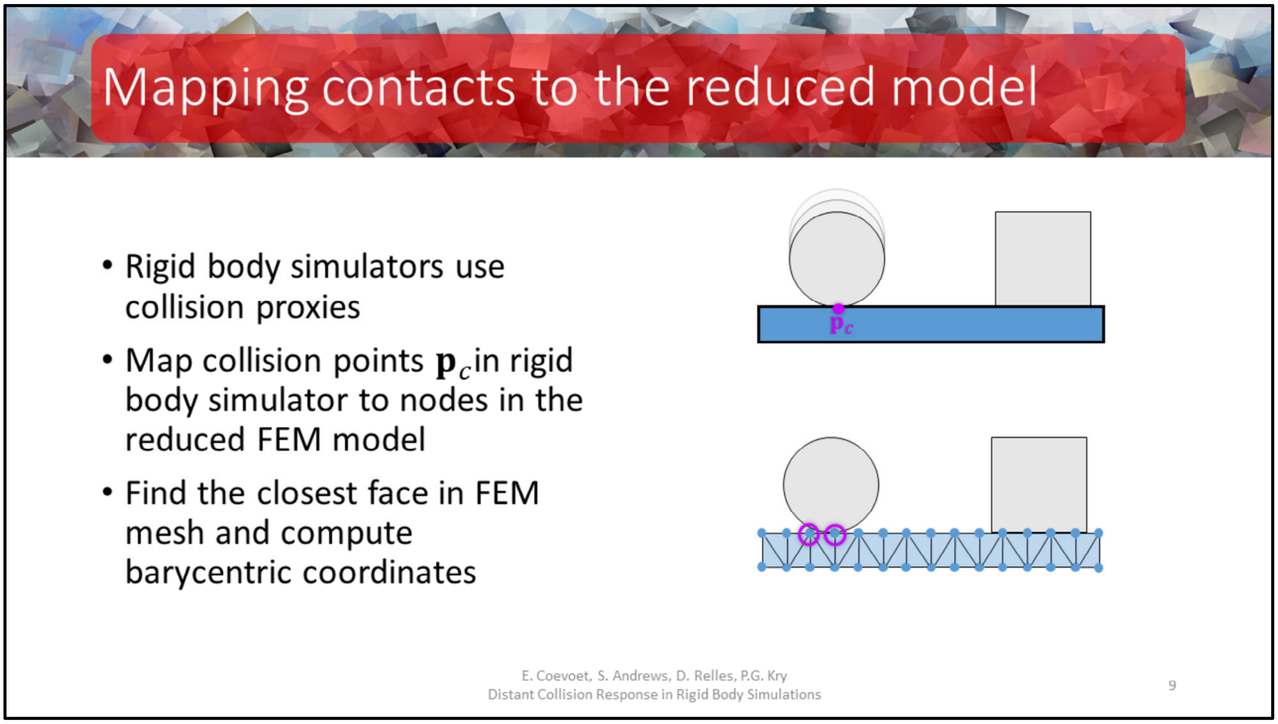
We then compute an impulse response at distant contact points based on a force loading obtained from the rigid body constraint solver, which causes a disturbance at distant bodies.

Let’s look at each of these steps is closer detail…




As a preprocess, we build an FEM model corresponding to distant collision objects in our rigid body simulations. By solving a generalized eigenvalue problem, we obtain a modal decomposition of the FEM mesh.

Only a subset of the modes are needed, and in our experiments we found that using the first 20 modes is suitable. This gives a reduced modal basis matrix that we use to map displacements and forces to and from the reduced model.




At runtime, rigid body simulations typically do not use FEM geometry for collision detection, but rather proxy geometries such as spheres, boxes, capsules, and convex hulls.

In order to compute a response used our elastic model, we therefore need to map impacting contact points from the rigid body simulation to the FEM mesh of our reduced model.

We query a copy of the mesh containing only surface nodes, in other words a triangle mesh, and this results in the barycentric weights that give the contact point in terms of the FEM nodal coordinates.

# Modal impulse

- Compute the reduced impulse by using modal basis $\mathbf{U}$ and matrix of barycentric coordinates $\mathbf{H}_c$ :

$$\mathbf{r}_c = \mathbf{U}^T \mathbf{H}_c^T \vec{\mathbf{n}}_c \lambda_c$$

with contact normal $\vec{\mathbf{n}}_c$ and constraint impulse $\lambda_c$

- Response to the loading $\mathbf{r}_c$ as a system of $m$ independent ODEs :

$$\ddot{\mathbf{q}}_j + 2\xi_j \omega_j \dot{\mathbf{q}}_j + \omega_j^2 \mathbf{q}_j = \frac{(\mathbf{r}_c)_j}{m_j} \qquad j = 1 \ldots m$$

E. Coevoet, S. Andrews, D. Relles, P.G. Kry
Distant Collision Response in Rigid Body Simulations

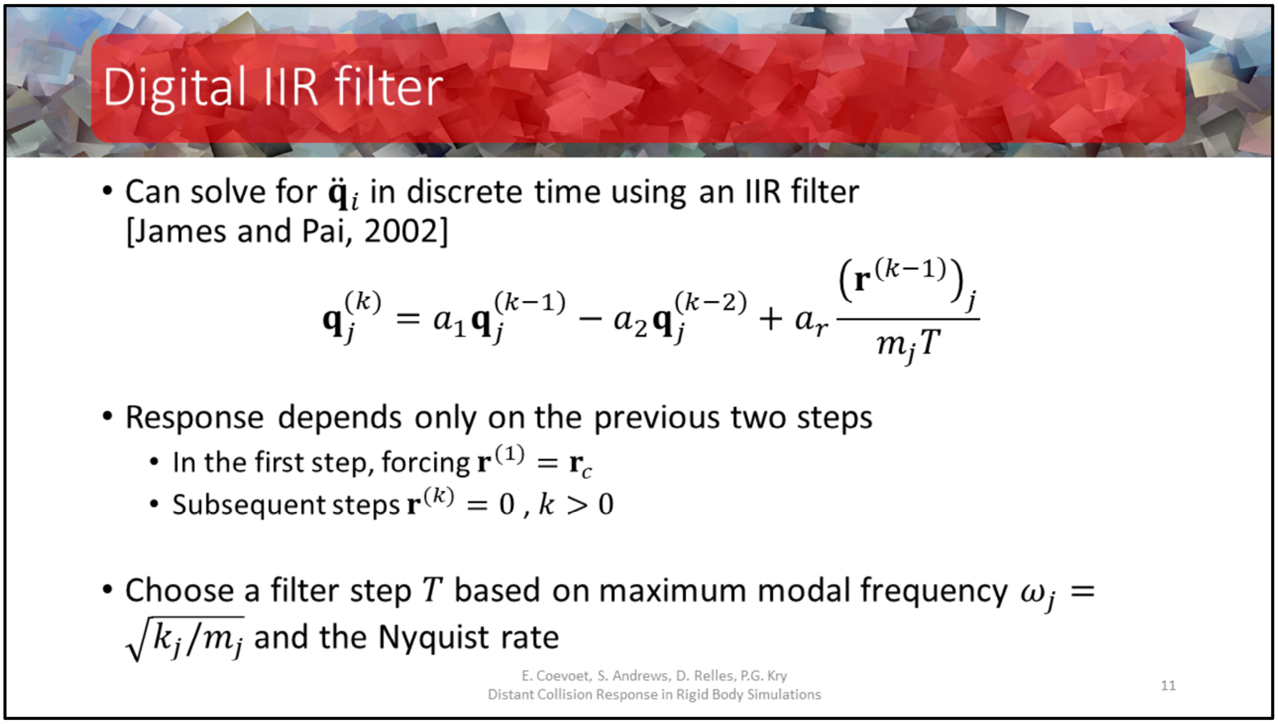
10

With barycentric weights in hand, we transform the impacting collision impulse from the rigid body simulation to the reduced elastic model.

The non-interpenetration impulse, lambda, that results from the collision is retrieved from the constraint solver and mapped to the reduced elastic model using a matrix of barycentric weights and the modal basis matrix.

This gives a force, or rather impulse, acting on the reduced model that we can use to compute a dynamic response in each mode due to the loading.

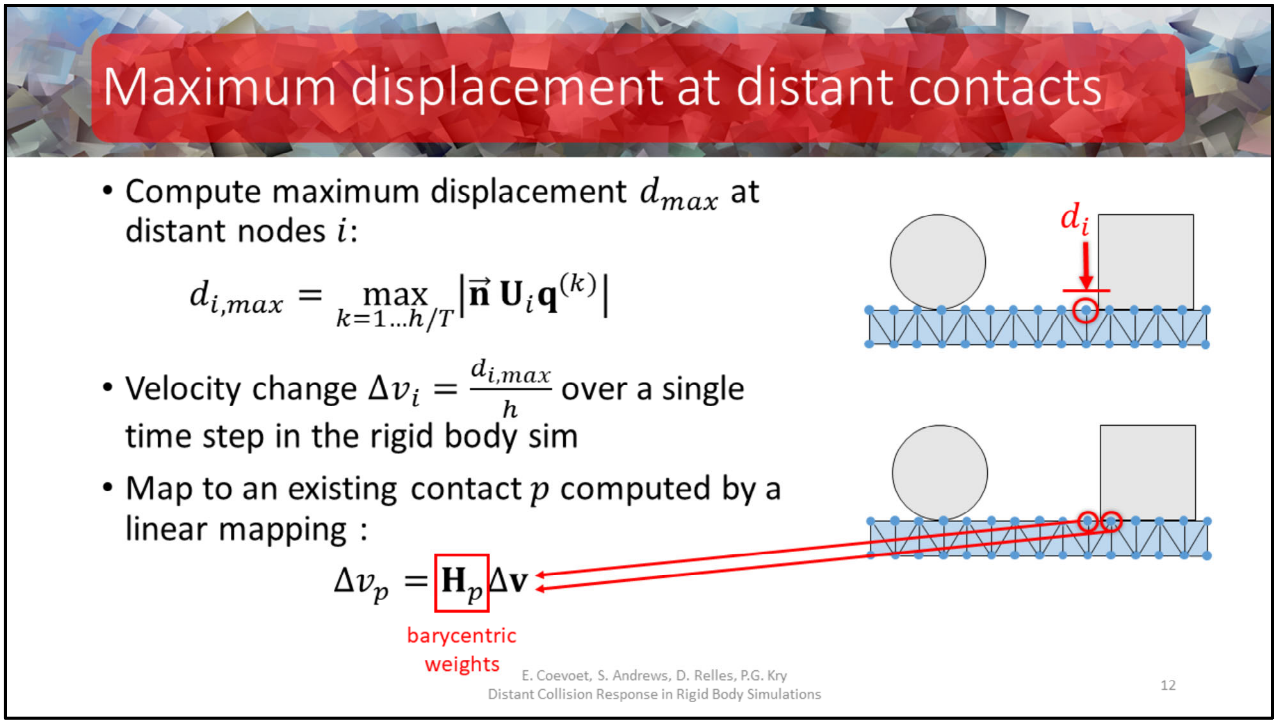
Conveniently, this can be done independently for each of the resulting differential equations, since each mode is decoupled in the reduced model.
I refer you to the paper for details on how to compute the coefficients shown in the ODEs here.




We can determine the response of each differential equation independently by using a digital IIR filter to integrate the equations. We compute the filter coefficients in the same way as James and Pai in their work on dynamic response textures.

Conveniently, even though the filter is evaluated over many steps. only the modal displacements, q, of the previous two integration steps and the loading function, r, are required to compute a new displacement value. Additionally, the loading function is only required for the first step, and in subsequent steps we set it to zero. Alternatively, we could apply the loading function over a small number of time steps to simulate a smoother transition.

The IIR filter update rate is based on the Nyquist rate of the maximum frequency across all modes, which can be high from stiff objects.
The filter time step, big T, is therefore several orders of magnitude smaller than the time step of the rigid body simulation, meaning that the digital IIR filter rate is many times that of the rigid body simulation. However, evaluating the IIR is [extremely] efficient.




Since the digital filter runs at a much higher frequency than the rigid body simulation, displacements computed by the IIR filter cannot directly be used to simulate a collision response.

Rather, we compute the maximum displacement over all steps of the IIR integration, which we run for a single time step of the rigid body sim. This displacement corresponds to the peak amplitude of a wavefront due to impact.

Then, velocity changes that result from the displacement are computed at existing contacts by linearly interpolating the velocity changes at nodal coordinates using the barycentric weights of the existing contact points.

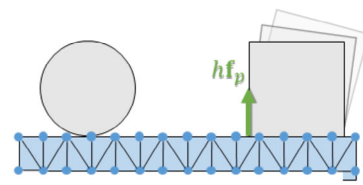
# Distant impulse response

- Apply a velocity change $\Delta v_p$ by an impulse in the normal direction $\vec{\mathbf{n}}_p$

$$h\mathbf{f}_p = m_{\text{eff}} \Delta v_p \vec{\mathbf{n}}_p$$

- Where the effective mass $m_{\text{eff}}$ is estimated using the non-interpenetration constraint Jacobian $\mathbf{J}_p$ :

$$m_{\text{eff}} = \frac{1}{\mathbf{J}_p \mathbf{M}^{-1} \mathbf{J}_p^T}$$





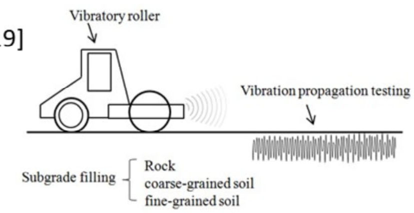

The velocity change is realized by applying an impulse at each existing contact.

The impulse is computed by estimating the effective mass at each contact point, which involves the contact Jacobian and the mass matrix of the rigid body simulation.

By taking into account the effective mass, the impulse is able to realize the desired velocity change at the next time step in the rigid body sim.

# Vibration in large objects

[Chen et al., 2019]




- For a source acting on the surface of terrain, vibrations propagate and attenuate according to a decaying exponential:

$$A_0 \exp\left(-\alpha(r - r_0)\right) \left(\frac{r}{r_0}\right)^{-\beta}$$

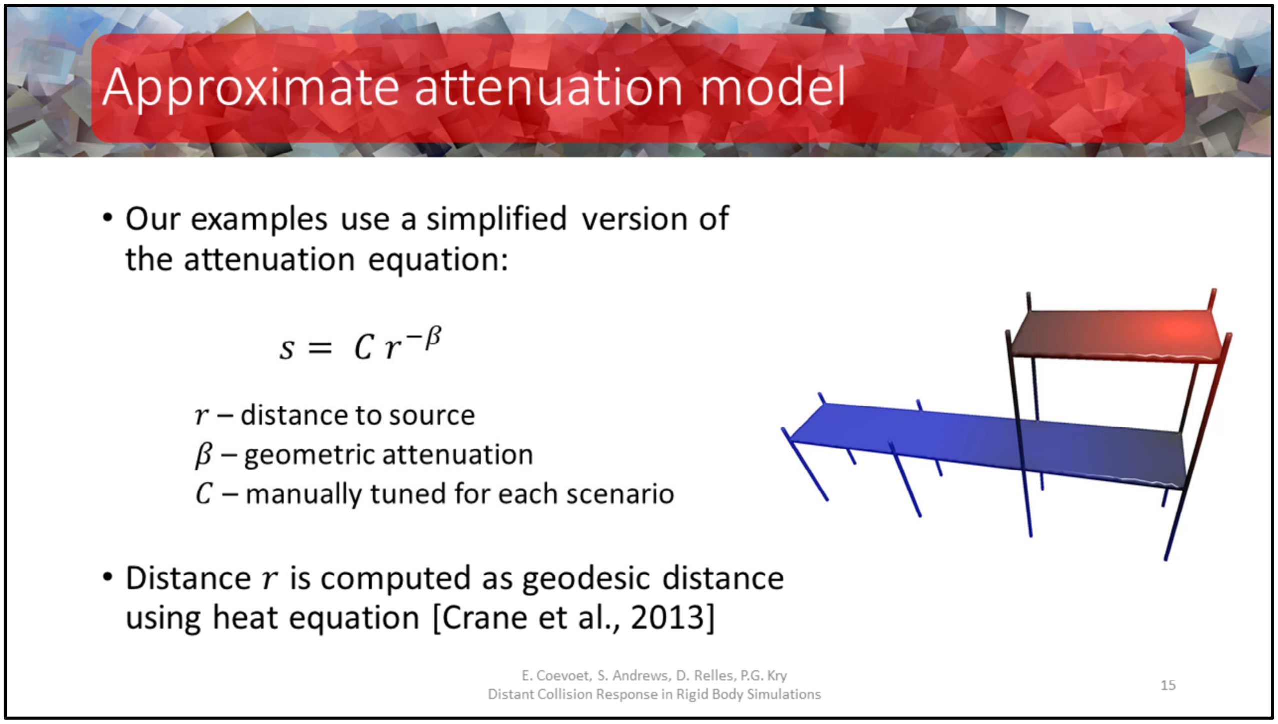
$A_0$ - amplitude at source
$r_0$ - minimum distance
$r$ – distance to source
$\alpha$ – material absorption
$\beta$ – geometric attenuation

E. Coevoet, S. Andrews, D. Relles, P.G. Kry
Distant Collision Response in Rigid Body Simulations

14

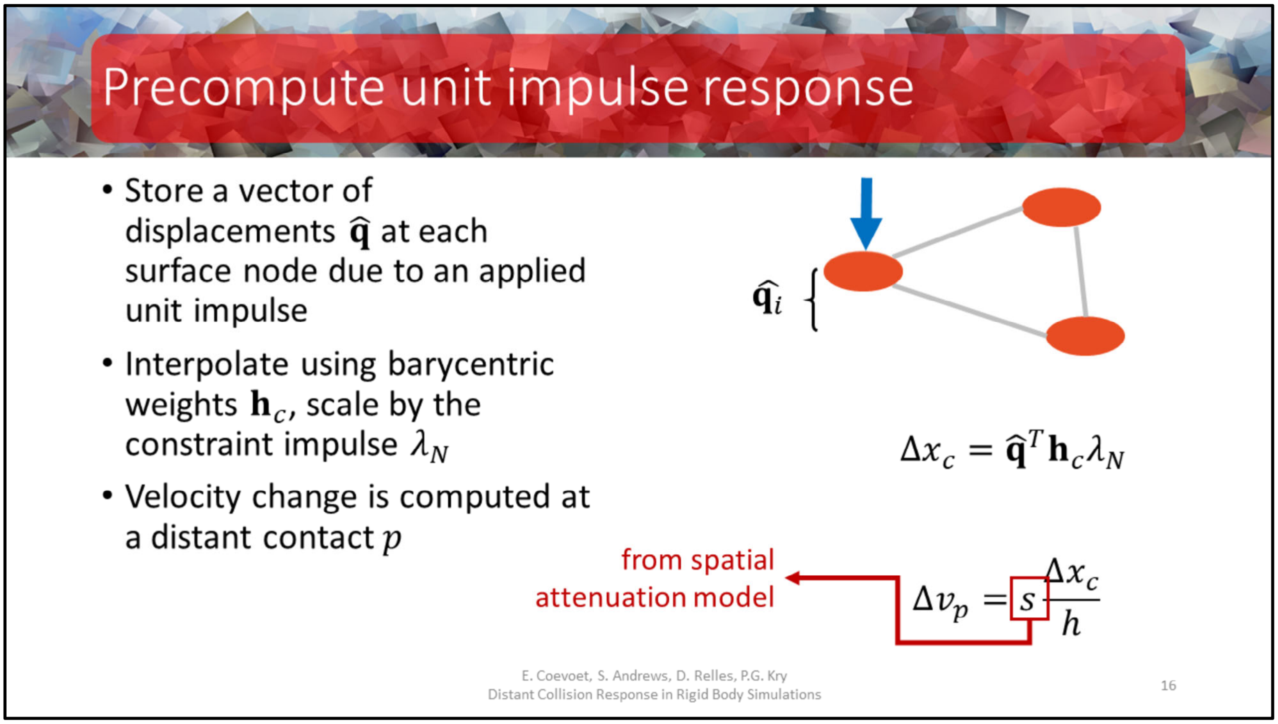
For large objects, such as terrain we observe that vibrations propagate as a traveling elastic wave.

If the source of vibration, such as an impacting collision, is located on the surface of the object, the vibrations will propagate and attenuate according to a decaying exponential. The amplitude at a remote contact is dependent on the initial amplitude at the source, the distance from the source, as well as several material properties. We therefore propose an alternative approach for computing the distant collision response for large-scale objects based on this model.




We found that a simplified version of the attenuation equation produced plausible behavior, while also being more straightforward to tune.
We compute a scaling of the amplitude due to material absorption by a constant parameter that is tuned to each specific scenario. Reasonable values for this parameter are discussed.
For the geometry term, we use $\beta$ = 0.5, 1.0, and 2.0 as the exponent term and observe that the response becomes more localized as the parameter is increased, which is expected given the role of the term to account for geometric attenuation of an elastic wave.

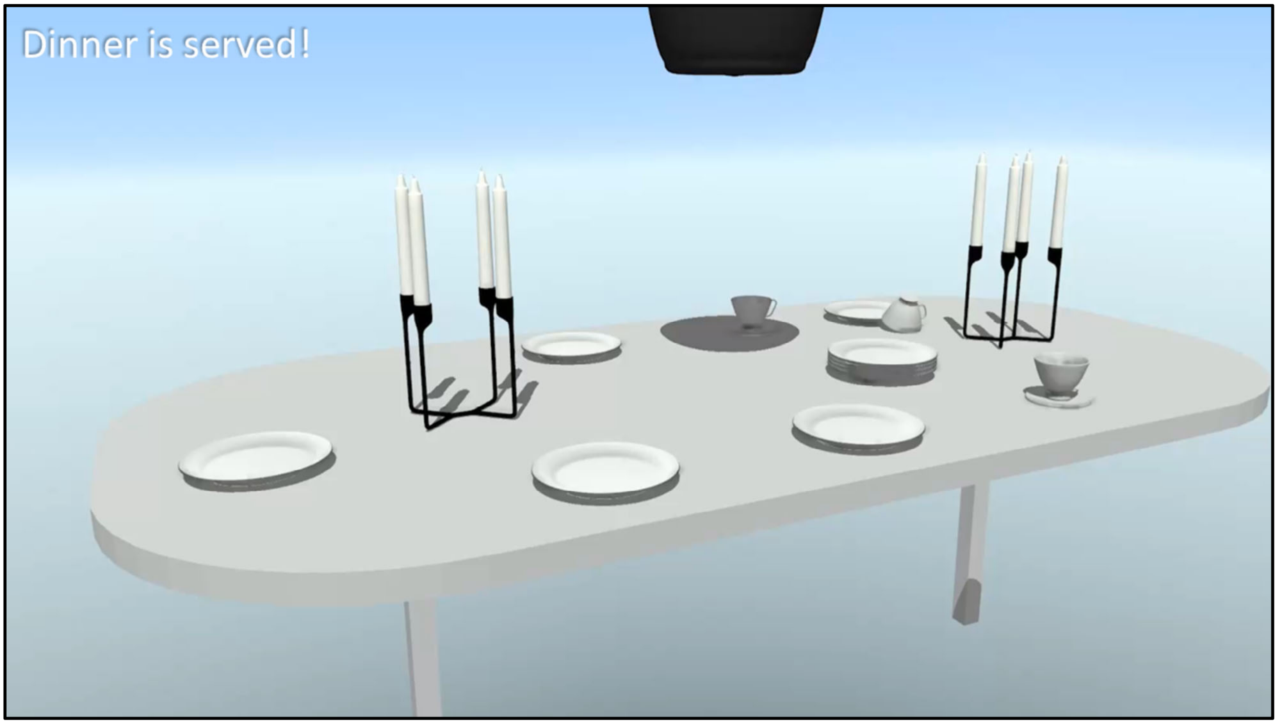
Since we are mainly interested in the effect of traveling waves observed at distant locations on the surface, we use a geodesic distance between points within an elastic model to compute a response




In order to estimate the initial amplitude of the elastic wave, we sample the displacement at each surface node by applying a unit impulse. An offline FEM simulation with small timestep is used for this purpose

Displacements are then retrieved by the rigid body simulation during an impact, and the amplitude is estimated by interpolating the displacement, q hat, across FEM nodes and scaling it by the impulse from the constraint solver.

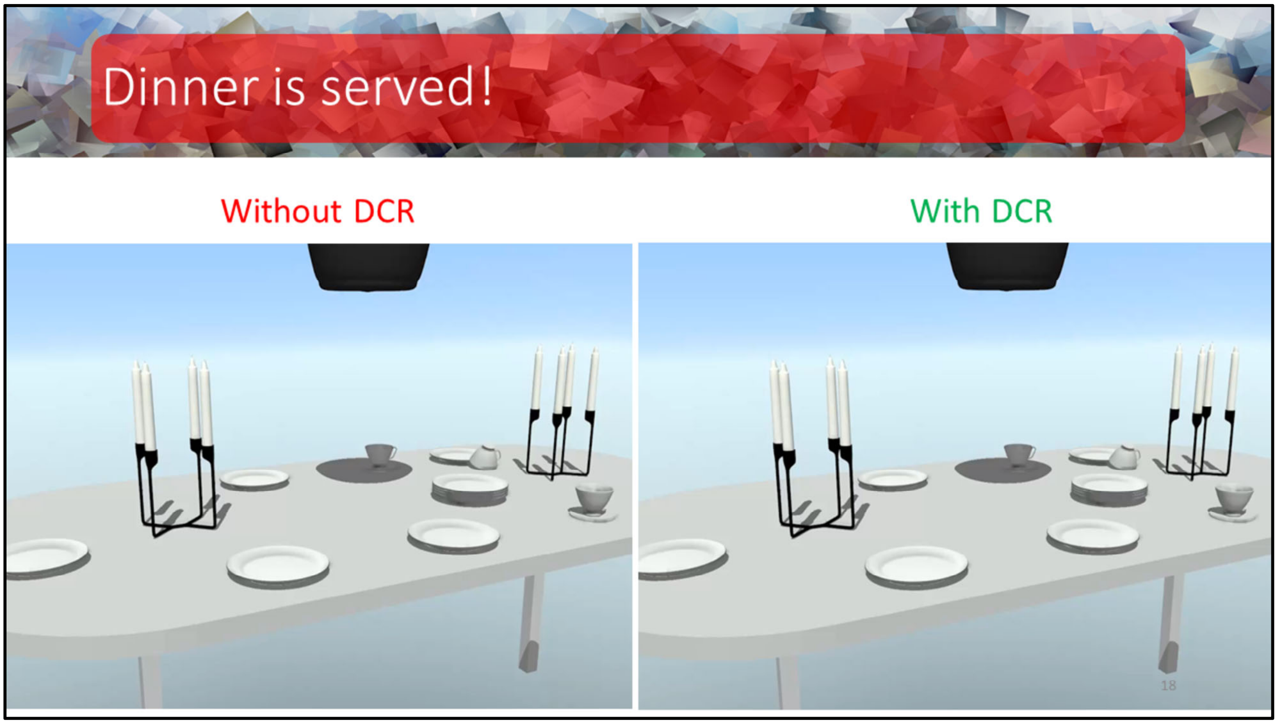
Then, to compute the amplitude at a distant contact, the displacement is again scaled by a factor computed from by the approximate attenuation model, and finally divided by the time step to compute a velocity change.




Now that you know the technical details, let’s see some results of distant collision response in action!

Here a heavy pot is dropped onto a table, causing plates and cups to jump. The collision response of the table is computed using a reduced elastic model.

Observe that even stacked objects exhibit plausible secondary dynamics even though they are not in direct contact with the table.



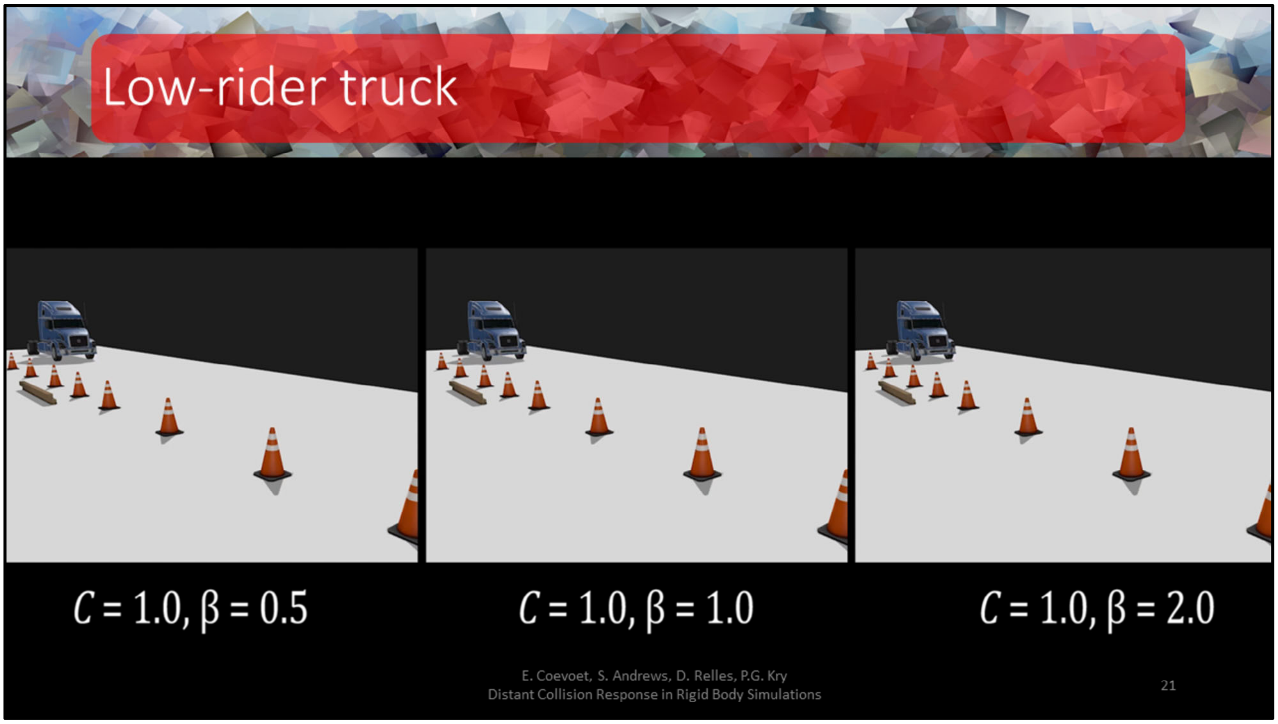

We have the same scene here shown with and without the DCR technique activated.

Using distant collision response, the scene is richer and more lively, with important dynamical cues about the impact being simulated.

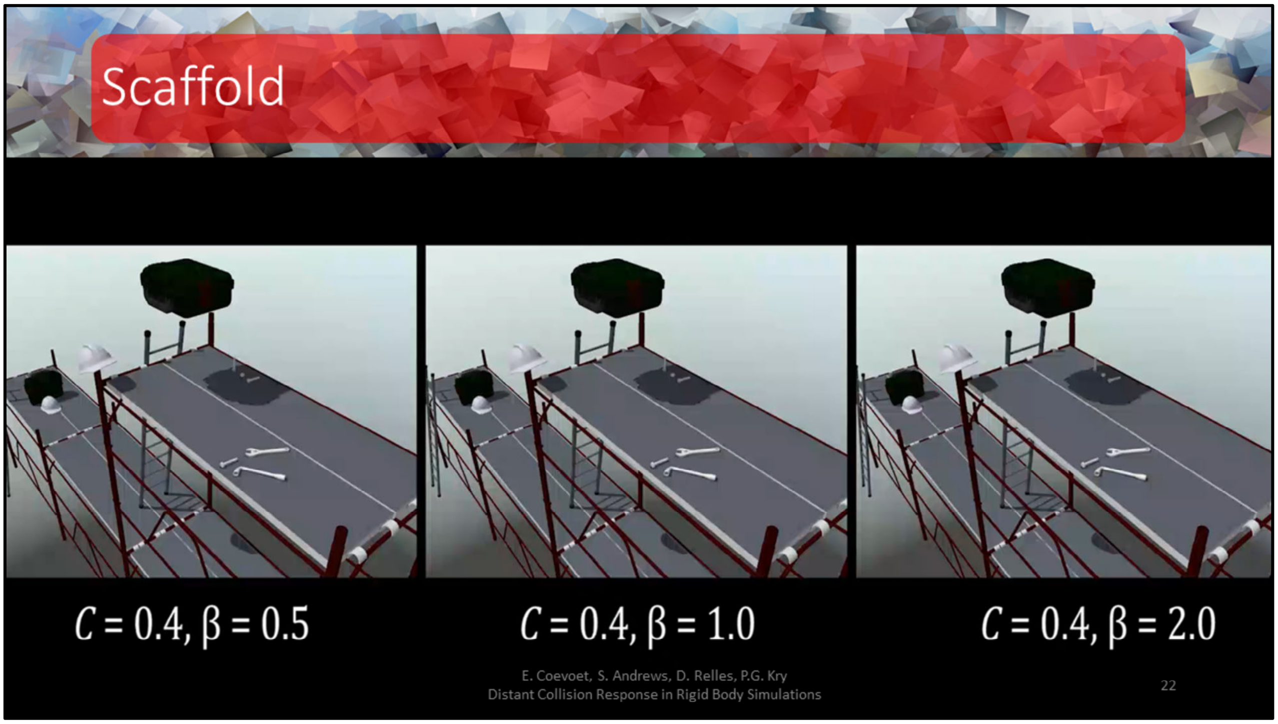

By comparing to the FEM simulation with small time steps, we see that distant collision response gives dynamical behavior that is qualitatively similar to the ground truth.


In this example, the cliff face is again modeled using an FEM mesh with precomputed modal response.

The impact of the truck hitting the base of the cliff causes the rocks perched above to come tumbling down.

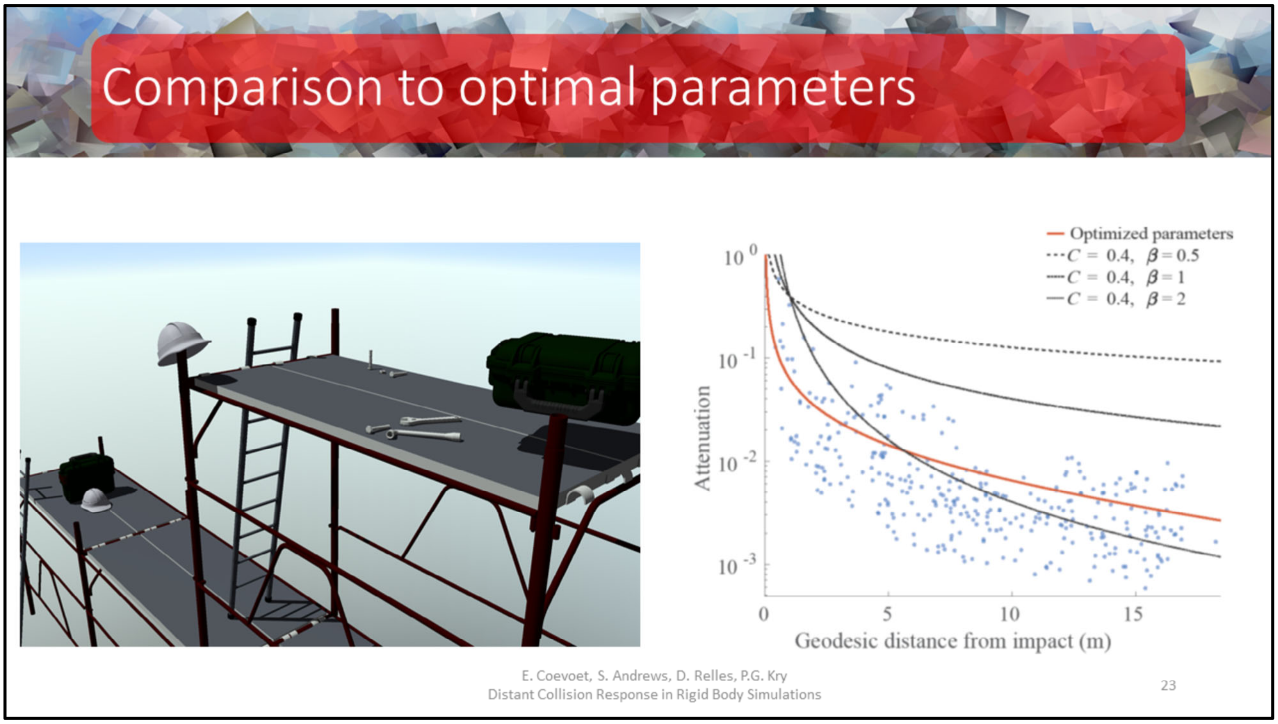



Here is an example using our spatial attenuation model. Observe that with an increasing value of beta, the area of effect of the computed disturbance is reduced.




We also tested the spatial attenuation model using a large scaffold example. The 'C' coefficient is tuned differently for this scenario, but the effect of beta is similar.

A geodesic distance is computed and cached for each resting contact point, which is used in the spatial attenuation equation to estimate the distance from the source of impact.




By optimizing for the parameters used in our spatial attenuation model using a data fitting technique, we observe that manually tuned values produce plausible results. For example, in the scaffold example, using values of C equal to 0.4 and beta equal to 2, the amplitudes are reasonably close to those measured in an offline FEM simulation.

## Performance

### Precomputed modal elastic model:

| Example | Model | # Vertices | # Triangles | # Modes | Time step | Time step (IIR) | # Distant contacts | Compute time |
|---|---|---|---|---|---|---|---|---|
| Dinner is served | table | 966 | 1928 | 20 | $10^{-2}$ | $4.9 \times 10^{-4}$ | 41 | 3.8 ms |
| Wrench on a roof | roof | 1002 | 2000 | 20 | $10^{-2}$ | $7.0 \times 10^{-3}$ | 7 | 0.8 ms |
| Scaffold | scaffold | 5384 | 10776 | 20 | $10^{-2}$ | $3.0 \times 10^{-3}$ | 58 | 27.5 ms |
| Low-rider truck | ground | 1570 | 3136 | 20 | $10^{-2}$ | $3.1 \times 10^{-4}$ | 40 | 5.6 ms |
| Rockfall | cliff | 7182 | 14360 | 20 | $10^{-2}$ | $4.0 \times 10^{-3}$ | 24 | 18.1 ms |
| Washing machine scene | floor | 360 | 716 | 20 | $10^{-2}$ | $1.7 \times 10^{-4}$ | 26 | 1.1 ms |
| Washing machine scene | fridge | 250 | 496 | 20 | $10^{-2}$ | $8.6 \times 10^{-5}$ | 6 | 0.2 ms |

### Spatial attenuation model:

| Example | Compute time |
|---|---|
| Scaffold | 30.6 ms |
| Low-rider truck (first) | 6.9 ms |
| Low-rider truck (subsequent) | 5.2 ms |

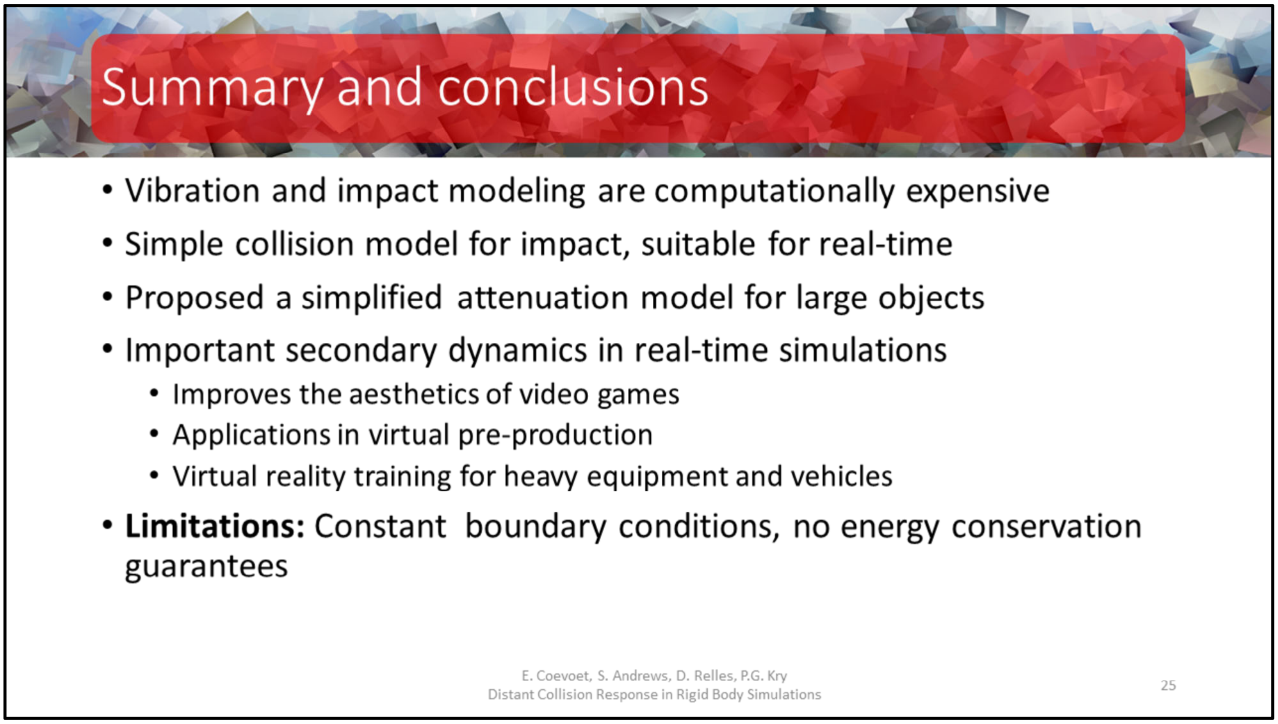
E. Coevoet, S. Andrews, D. Relles, P.G. Kry
Distant Collision Response in Rigid Body Simulations

24

Finally, we demonstrate that our approach runs at interactive or real-time framerates by reporting performance details for our implementation.

The compute time using the reduced elastic model increases with complexity of the FEM mesh and number of distant contact. However, we observed that determining the barycentric weights tends to dominate the overhead, and we believe an optimized or parallelized implementation of this function could significantly reduce the compute time.

Similarly, the spatial attenuation model was tested using the scaffold and low-rider truck example, and compute times are comparable. However, in addition to the barycentric weights, the computation of geodesic distances introduces additional overhead related to the solution of the heat equation. Caching distance values of resting contacts helps, and we do this in a lazy fashion, which is why we report the first impact and subsequent impacts in the truck example separately.




To conclude, the true problem of computing vibrations due to collision is very messy! Simplifications are necessary to have a solution that can be computed at real-time or interactive framerates.

We have proposed a simple collision mode that gives a plausible response at distant contact points when impacts occur, and the approach is suitable for real-time simulations.

Furthermore, we have proposed a simplified model for large scale objects with attenuation of traveling elastic waves.

DCR provides important dynamical cues that are missing from many real-time simulations. We envision that our approach can be used in video games, virtual pre-production, and virtual reality training to improve the plausibility of secondary dynamics in these applications.

However, the approach does have some limitations. Specifically, our approach ignores boundary conditions in order to simplify computation of the reduced elastic model, but which could ultimately change the elastic modes. This is a large approximation, but is realized in a physically based manner.

Furthermore, there are no guarantees regarding energy conservation with our approach. However, we believe this would be an interesting consideration for future work.


Finally, we graciously acknowledge NSERC and CMLabs for funding and supporting this research.

Thanks for watching!