

EFFECT OF ADDED MASS ON SUBMERGED VIBRATED PLATES

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ABSTRACT

Vibration of solid-fluid interaction systems requires a deterministic knowledge of vibration analysis and structures coupling. It is evident that a structure vibrating in a fluid has to support an interactive inertial effect caused by the fluid which changes the dynamic behavior of the solid itself. This paper presents an experimental study on the effect of added mass on submerged plate structures which are subjected to vibration. Different plates vibrating in water at different depths with various geometry ratios are studied using a number of boundary conditions. Dynamic testing is conducted in time domains to identify the modal parameters of the structures. The modal added mass value and added mass coefficient are then derived from the change of modal frequencies from air to water. This is followed by a comparison of experimental results with several numerical analyses found in the literature.

1. INTRODUCTION

In solid-fluid interaction study, plate structures have often been considered for two major reasons. Firstly, plates are widely used in common structures for ships, heat exchangers, aerospace and civil engineering. Secondly, the plate has an advantage in terms of structural convenience in modeling, analysis and testing. From a theoretical point of view, there are many published articles describing developments for dynamic analysis of plate structures. Lamb [1] conducted what is considered the first study on frequency analysis of a plate-fluid contact system using the Rayleigh method. Fu and Price [2] studied vibration responses of cantilevered vertical and horizontal plates partially or totally immersed in fluid. They assumed that the plates vibrated in a semi-infinite fluid medium. A finite element method and a singularity distribution panel approach were used to analyze the dynamic responses of plates in air and also to determine the hydrodynamic coefficients for each element in contact with fluid. Haddara and Cao [3] investigated dynamic responses of plates in air and submerged in fluid under various boundary conditions. They presented an approximate solution for the equation of motion of a plate coupled with fluid and provided an analytical added-mass factor depending on the height of the free surface and the depth of fluid under the plate. An approximate expression for the evaluation of the modal added mass was derived. Their results were in agreement with Fu and Price; when the plate depth attains 25% of its length, first frequency bending starts to change. Liang [4] took an alternative approach, using the Rayleigh Ritz method to derive the added mass factor for each mode of cantilever plate vibration with a correction coefficient for the aspect ratio. These added mass factors were then used to calculate free vibration of the plate. Use of the finite element method for structural analysis is well known, however the problem of solid-fluid interaction is so complicated that, to date, no coupled element has been developed to model this situation in commercial software. Recently, a research group at Polytechnique de Montreal developed a hybrid finite element model using the Sanders' thin plate method to analyze the vibration of a variety of classes of plates in fluid. The interactive effects are distinctly derived and include

inertial effects resulting in a pressure expression whose product with the structural shape function produces a virtual added mass schematic [5, 6].

In the field of experimentation, which can be considered as practical research to validate theoretical models and applications, several separate tests on concrete structures are found in the literature. Powel and Robert [7] were the first investigators to experiment with circular bound fixed plates in contact with fluid. Their observations reached slightly higher values than those presented by Lamb [1]. Lindholm et al. [8] carried out an extensive experimental study of the response of cantilever plates in air and in contact with fluid. The plates with different aspect ratios and thicknesses were horizontally and vertically placed or inclined. The results were compared with theoretical predictions based on simple beam theory, thin plate theory and chord-wise hydrodynamic strip theory. An empirical correction factor was introduced to achieve good theoretical and experimental correlation. It was concluded that the added mass factor changed with submerged depth of the plate, but significant change occurred only when the submerged depth was less than about one half the span length of the plate. Haddara and Cao [3] also reported a validation of analytical analysis using experiments on a CFCF and a SFSF plate submerged in water. Natural frequencies and damping rates were identified using a Fourier transform technique. Several important results can be cited as follows: *i*) the change of the first bending frequency mode was about 31% to 32%; *ii*) the added mass factor seemed to increase steadily until the depth of submergence reached about 10% of the length of the plate, then it leveled off to a steady value; *iii*) added mass value was inversely proportional to the natural frequency, except for the torsion modes which had same added mass factor and *iv*) there was a sharp increase in damping when the plates were submerged in water, for all modes. The torsion modes exhibited the smallest increase.

It can be seen that a notable disagreement exists between experimental and theoretical results. Since the identification method used in all of the above experiments is the simple one-peak picking method based on Fourier transformation, the result is a high variance of identified frequencies and some low excited modes can be missed. This paper is a follow up to a previous article [9] and uses an innovative identification method on numerous plates to review the case of vibration of plates in water and accurately identify the modal parameters of the system and derive the modal added mass factor for each mode. This research can also be seen as a validation of the hybrid finite element model developed earlier [5].

2. REVIEW OF ADDED MASS EFFECT

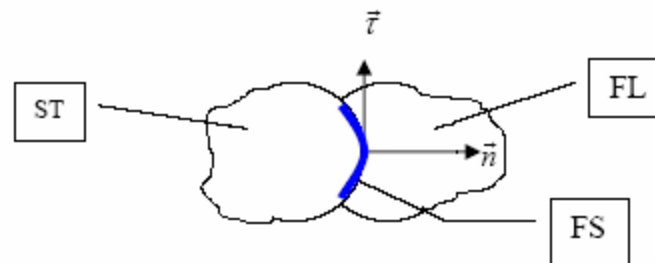


Fig. 1 Solid-fluid interaction

Consider a structure (ST) and fluid (FL) with an interaction surface (FS). At an arbitrary point, a normal n and tangent t can be identified as shown in Figure 1. In a vacuum, the structure has its own orthogonal modes $X_i(r)$ which correspond to ω_i natural frequency and m_i generalized mass.

Displacement of the structure can be written using modal bases and a vector of modal participant factors $a_i(t)$:

$$x_s(r, t) = \sum_i a_i(t) X_i(r) \quad (1)$$

When the structure vibrates in a static fluid, the linear influence can be considered as a force F_g on the structure. So,

$$M\ddot{x}_s(t) + C\dot{x}_s(t) + Kx_s(t) = F_g \quad (2)$$

If we project on the i^{th} mode, one can obtain:

$$[m_i] \ddot{a} + i\omega_i [c_i] \dot{a} + \omega_i^2 [m_i] a = [F_{gi}] \quad (3)$$

This force F_{gi} is originally from the inertial effect of fluid around the structure, so the pressure on the structure is:

$$p(r, t) = -\sum_j \ddot{a}_j(t) p_j(r) \quad (4)$$

The generalized force F_g on the whole interaction surface can be obtained:

$$F_{gi} = \int_{FS} p(r, t) X_i(r) n dFS = -\sum_j \ddot{a}_j(t) \int_{FS} p_j(r) X_i(r) n dFS = -\ddot{a} m_{ai} \quad (5)$$

Then

$$[m_i] \ddot{a} + i\omega_i [c_i] \dot{a} + \omega_i^2 [m_i] a = -\ddot{a} [m_{ai}] \quad (6)$$

Or

$$([m_i] + [m_{ai}]) \ddot{a} + i\omega_i [c_i] \dot{a} + \omega_i^2 [m_i] a = 0 \quad (7)$$

It is evident that the effect of the fluid has modified the mass matrix in the dynamic equation. The matrix m_a is called the matrix of added mass:

$$m_{ai} = \int_{FS} p_j(r) X_i(r) n dFS \quad (8)$$

With the presence of added mass, one can immediately affirm that:

- ✓ The total mass of the system increases,
- ✓ The natural frequencies of the system decrease.

The effect of added mass can now be calculated. We can observe from Equation (8) that this effect depends on the density of the fluid, the geometry of the structure, its mode shapes and its vibration amplitude. This effect is not influenced by fluid flow velocity or by the fluid's viscosity. In the case of still water, only the effect of added mass is considered. The matrix of added mass is always symmetric and definitely positive.

3. STRUCTURES TESTING

To provide a general case, two types of plates with different dimensions were studied to measure their vibration responses (Figures 2 and 3). The tests were conducted both in a vacuum and in a water tank at different depths in order to consider the effect of added mass. All three plates are steel, each weighing 7872 kg/cm³, the elastic modulus is 2e11 Pa and the Poisson ratio is 0.29. Plate thickness is constant at 1.905mm. Table 1 shows the configuration and disposition of eight accelerometers on each plate.

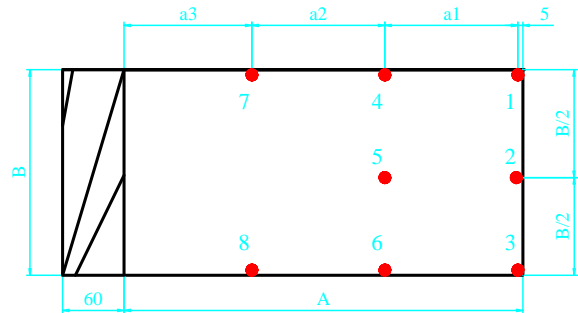
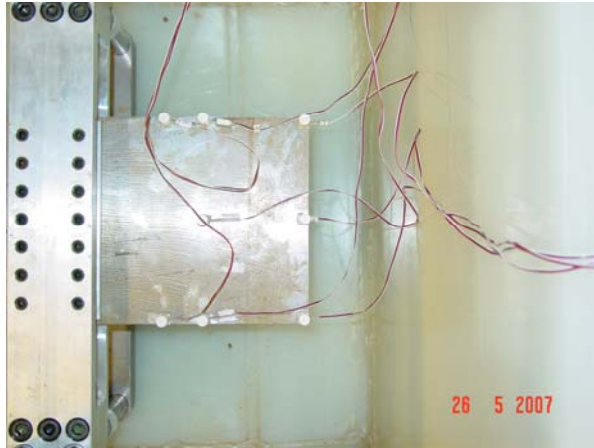


Plate	A(mm)	B(mm)	a1(mm)	a2(mm)	a3(mm)
1	377	201	150	150	72
2	201	201	95	40	60
3	106	201	45	30	26

Fig. 2 Plate CFFF test configuration

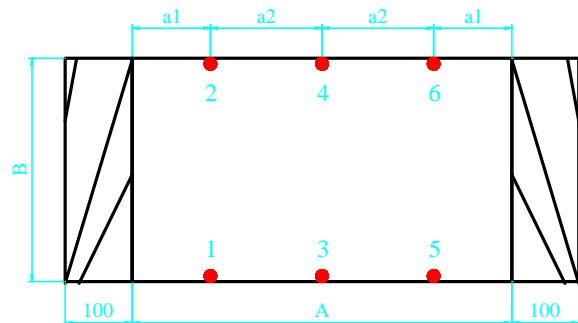
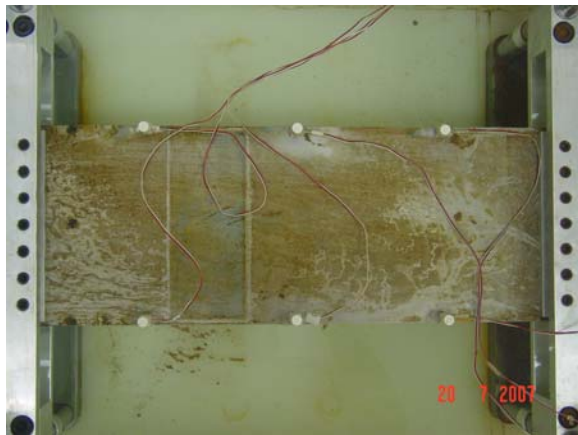


Plate	A (mm)	B (mm)	a1 (mm)	a2 (mm)
4	500	201	100	150
5	201	201	50	50

Fig. 3 Plate CFCF test configuration

Since we are not concerned about the excitation, an impact hammer was used to excite the vibration of the plate without any measurements of force.

4. IDENTIFICATION METHOD

The modal parameters of the plate were identified via a time domain method using a multivariable autoregressive model simultaneously on all sensor response data. Least squares were used to estimate the model parameters and were implemented with QR factorization which gives an unbiased and fast computation. The method is described more detail in a companion paper, see [9].

5. RESULTS AND DISCUSSION

5.1 Agreement between analyzed and identified values

To view the accuracy of the identification method the reader can refer to the companion paper which has a critical assessment of the method. For these concrete plate structures one can compare the results of the first five identified frequencies with those identified using finite element analysis (FEA) of plates in a vacuum (Table 1; (1) is the identified value and (2) is the FEA value).

Table 1: Identified and analyzed frequencies

Plate	Mode 1 (Hz)		Mode 2 (Hz)		Mode 3 (Hz)		Mode 4 (Hz)		Mode 5 (Hz)	
	(1)	(2)	(1)	(2)	(1)	(2)	(1)	(2)	(1)	(2)
1	39.80	40.04	92.30	98.57	225.50	246.42	284.00	313.49	318.00	358.12
2	10.29	11.18	44.21	45.75	67.58	69.75	145.01	149.85	187.63	195.67
3	6.20	6.33	33.14	33.33	39.40	39.57	104.80	106.14	110.00	111.29
4	38.59	40.64	74.61	76.76	107.77	112.19	163.80	169.79	208.23	220.76
5	233.95	256.11	309.40	305.00	478.54	499.75	608.57	707.89	735.86	775.93

5.2 Agreement between repeated tests

Consider plate number 4 results for three different testing times. The first test was done with a considerable excitation force, whereas the two other tests use a reasonable force value. All tests were excited with hammer impacts at the same point. On Figure 4 one can see a discrepancy between different testing times. During the first test with the largest excitation force, all identified frequencies were lower than those of the two other tests whereas the damping rates and mode shapes stabilize at the higher values. While the discrepancies of frequencies and mode shapes are negligible, a noticeable variation is found on damping rates with the amount up to 50%.

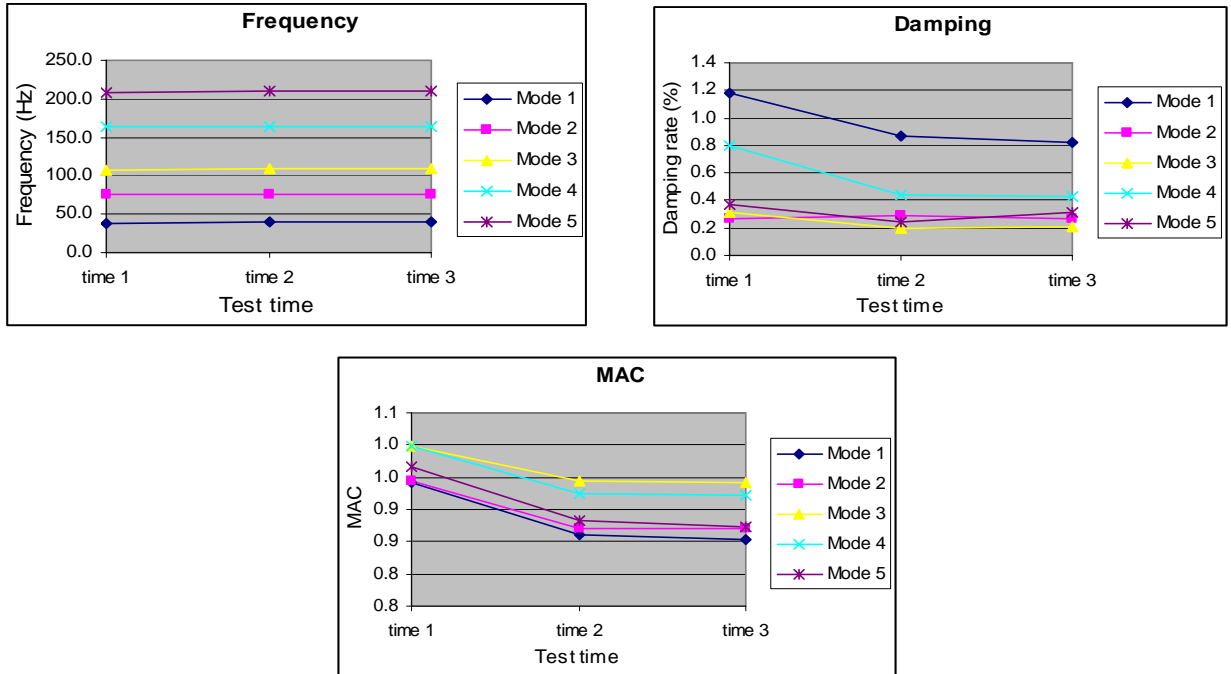


Fig. 4 Three testing times (CFCF A500B201)

5.3 Effect of frequency change and added mass

Each plate is submerged in the water at different depths and the depth/length ratio varies from 0 (in a vacuum) to 0.6 (totally submerged). The diagrams below show the variation of several first frequencies with the position of the plate in each case.

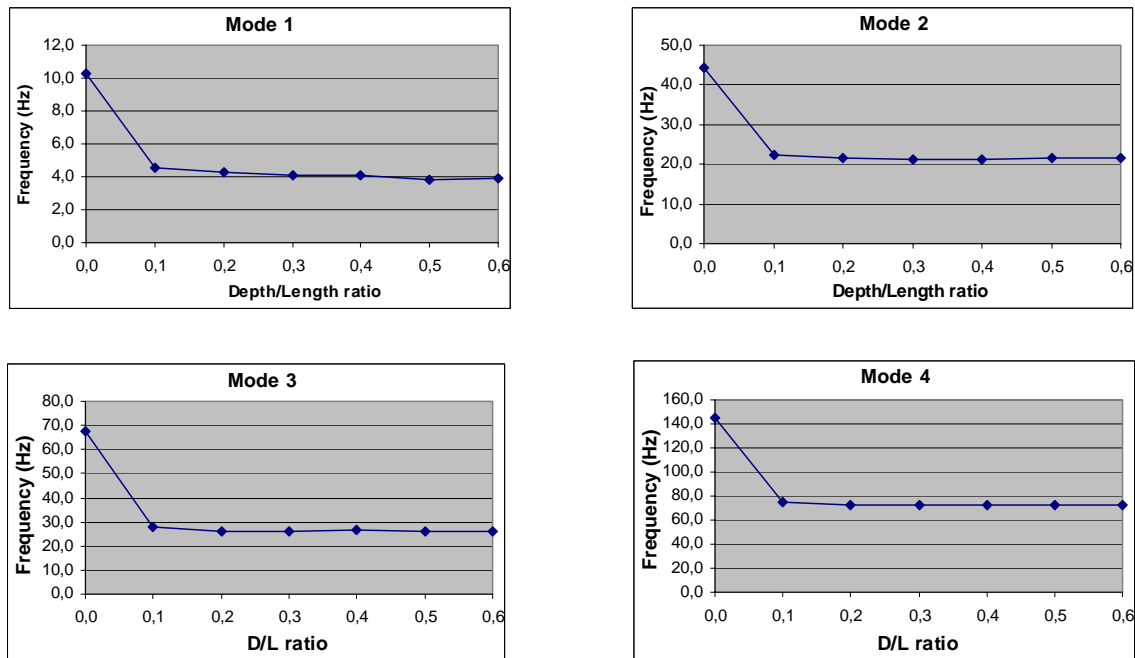


Fig. 5 Frequency change of plate 1 (CFFF A377B201)

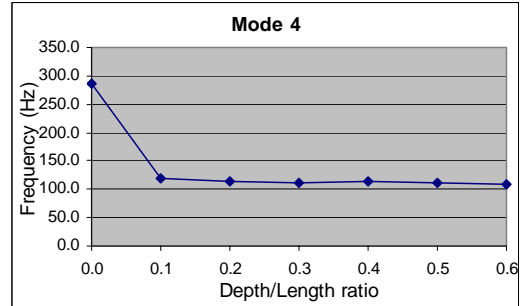
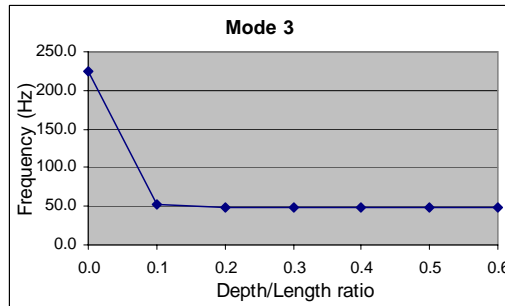
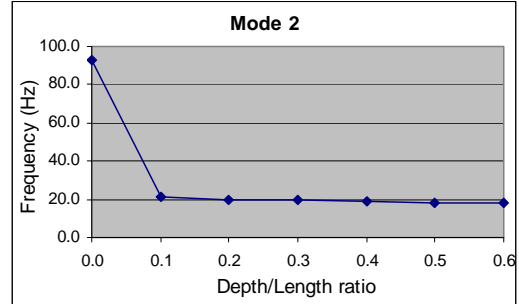
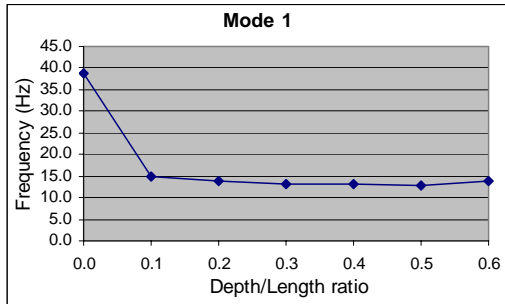


Fig. 6 Frequency change of plate 2 (CFFF A201B201)

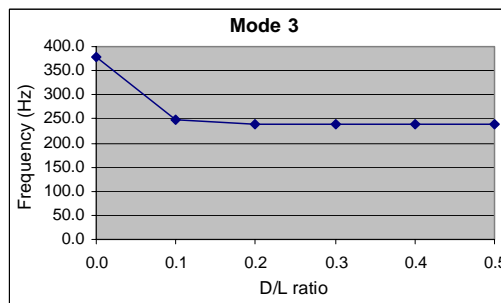
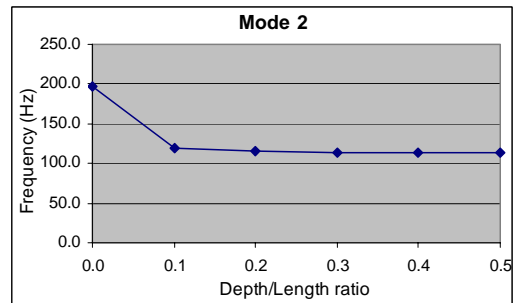
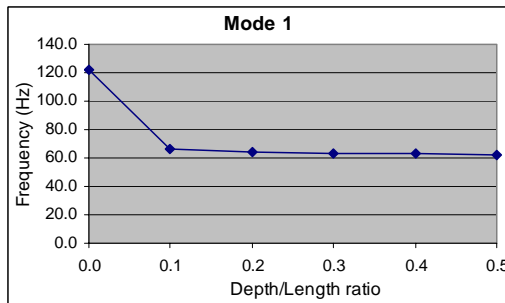


Fig. 7 Frequency change of plate 3 (CFFF A106B201)

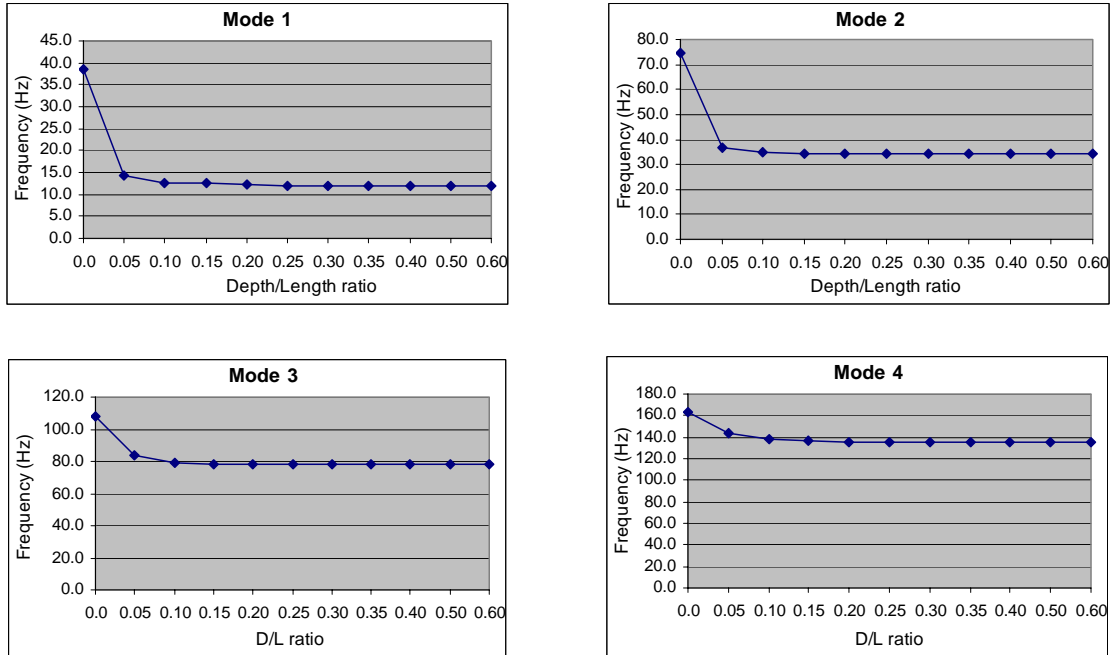


Fig. 8 Frequency change of plate 4 (CFCF A500B201)

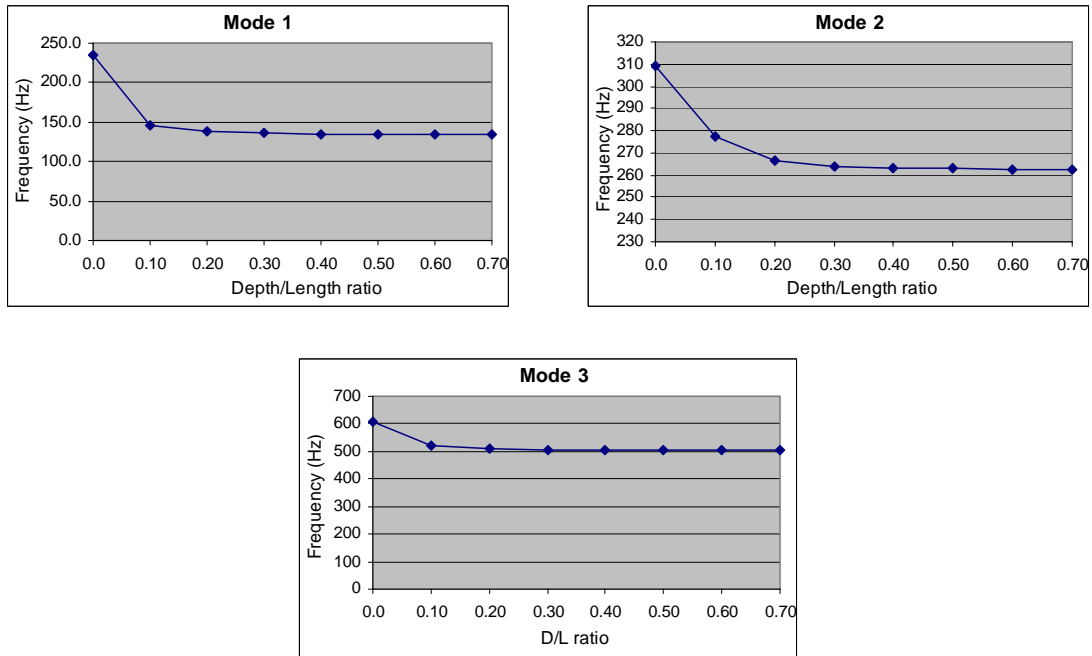


Fig. 9 Frequency change of plate 5 (CFCF A201B201)

It can be seen from these diagrams that every frequency mode decreases when the plate is submerged in water. This reduction is relevant even at low depths of 0.1 times the plate's length. When the plate is submerged, the frequencies change slightly as the depth varies and all cases are considered to cease at a depth/length ratio of 0.5. The frequency change effect is most significant for the first bending mode. For a square CFFF plate this largest value of frequency change is up

to 62% .The frequency change value is 69% for a rectangular CFCF plate, greater than the value Haddara et Cao provided earlier. It seems that the lower the frequency value, the greater the frequency change except at torsion modes where the change in frequency value is nearly identical. This result agrees well with the conclusions of Haddara et Cao.

To calculate the added mass factor, we consider that the structure’s rigidity is retained constant when it is submerged in fluid. The change of frequencies thus exhibits only an affect of added mass on the structure. Therefore one can have:

$$\begin{aligned} K_v &= \omega_{n,v}^2 M_v = K_f = \omega_{n,f}^2 M_f = \omega_{n,f}^2 (M_v + M_a); \\ C_v &= 2\xi_v \omega_{n,v} M_v; \\ C_f &= C_v + C_a = 2\xi_f \omega_{n,f} M_f \end{aligned} \quad (9)$$

Where K_v , M_v , C_v , $\omega_{n,v}$, ξ_v and K_f , M_f , C_f , $\omega_{n,f}$, ξ_f are the orthogonal modal rigidity, modal mass, modal damping, natural frequency and damping rate of a mode in a vacuum and fluid respectively. M_a denotes the modal added mass of this mode.

From equation (9), one can derive the added mass and added damping factor

$$\frac{M_a}{M_v} = \frac{\omega_{n,v}^2}{\omega_{n,f}^2} - 1; \quad \frac{C_a}{C_v} = \frac{\xi_f \omega_{n,v}}{\xi_v \omega_{n,f}} - 1 \quad (10)$$

From this derivation the following are diagrams of added mass versus the depth/length ratio (D/L) of the plate in water.

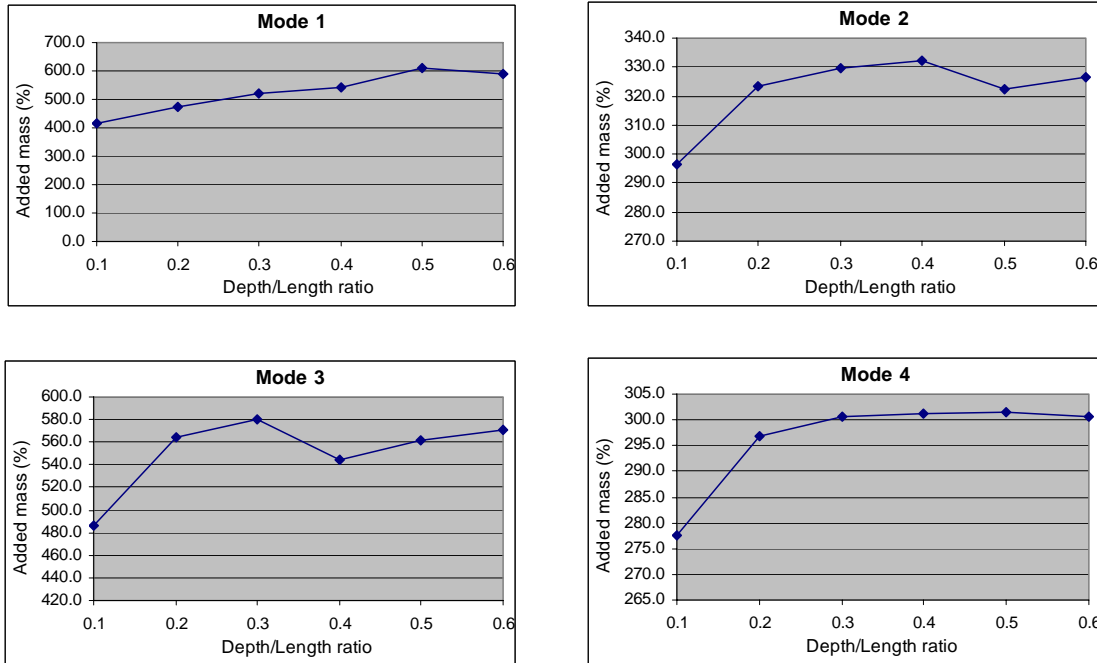


Fig. 10 Added mass on plate 1 (CFFF A377B201)

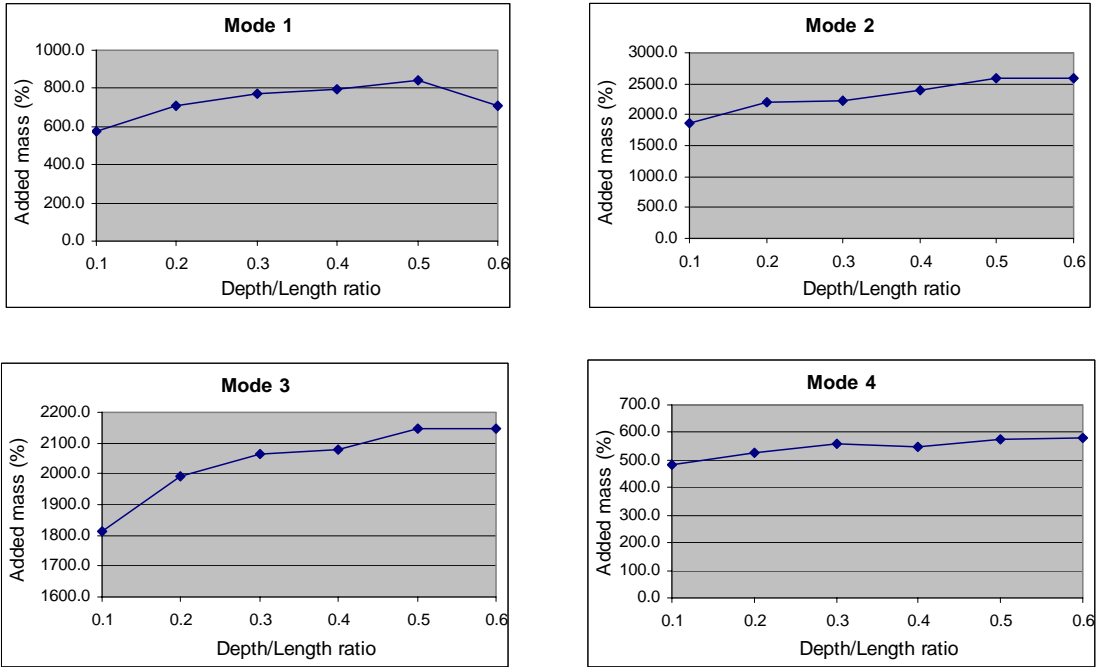


Fig. 11 Added mass on plate 2 (CFFF A201B201)

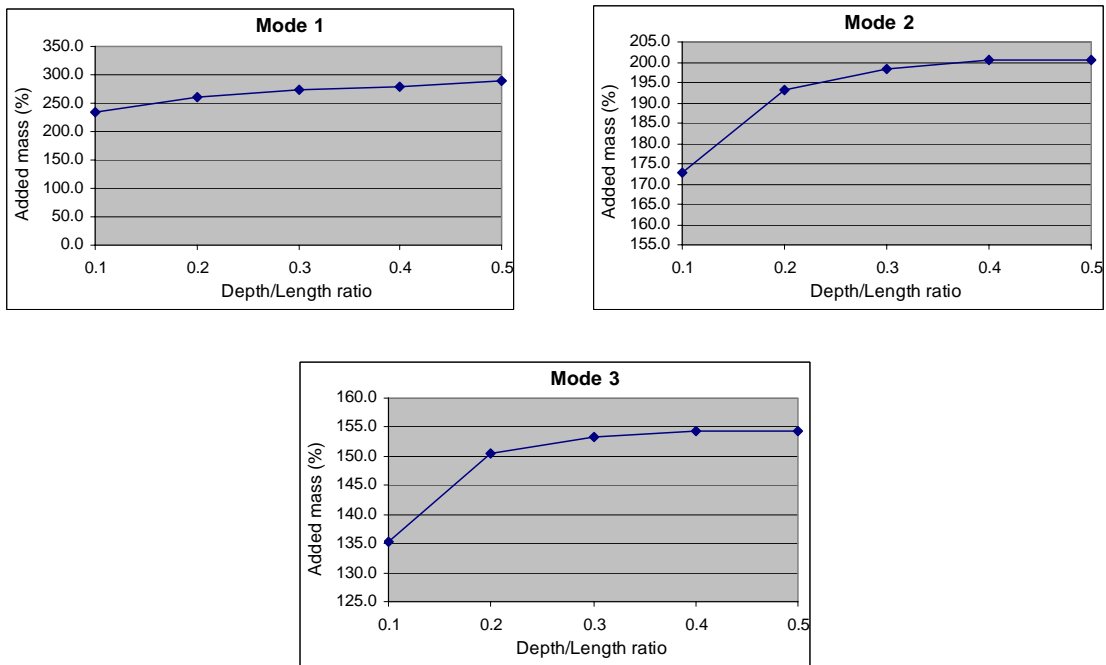


Fig. 12 Added mass on plate 3 (CFFF A106B201)

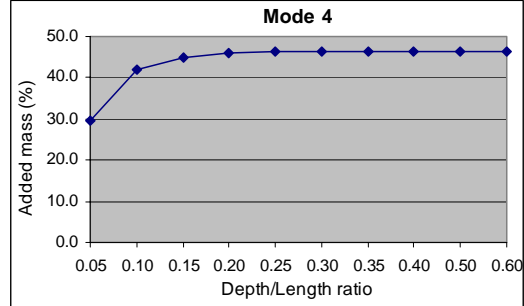
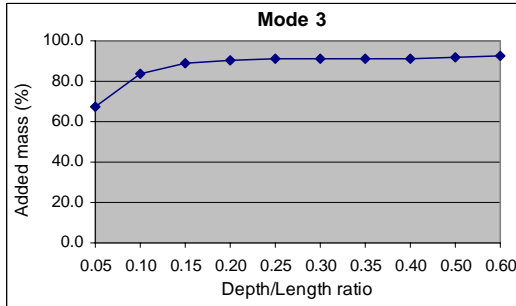
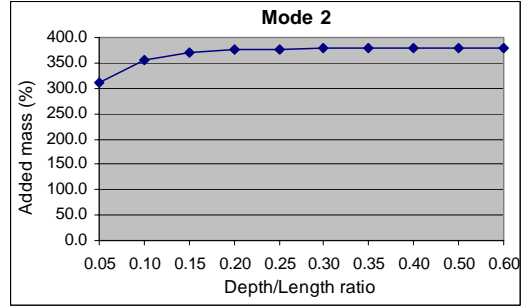
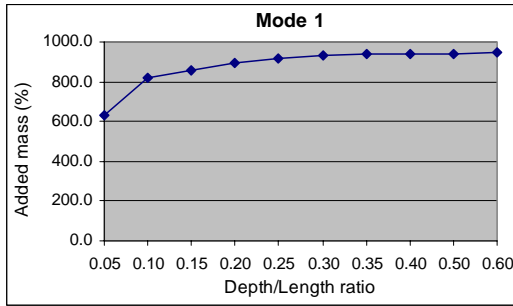


Fig. 13 Added mass on plate 4 (CFCF A500B201)

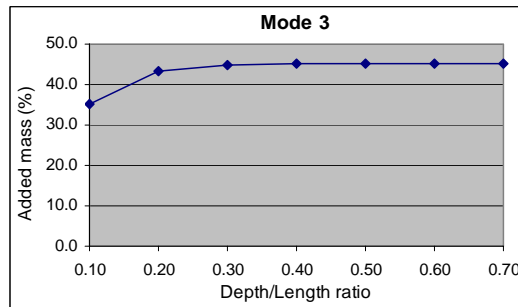
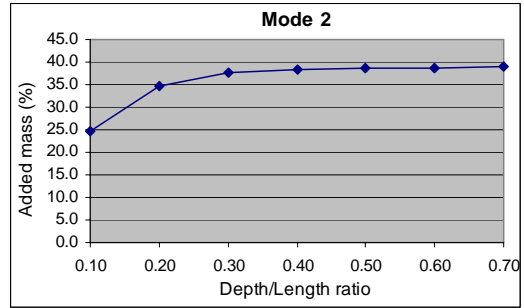
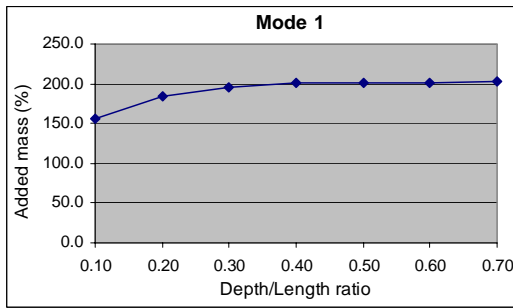


Fig. 14 Added mass on plate 5 (CFCF A201B201)

It is evident from Figures 10 to 14 that the added mass factor is different for each mode and is most significant for the lowest frequency mode exhibiting a mass value more than ten times greater than the mass of this mode in air. Therefore this first mode of vibration is the most important one in submerged plate vibration. The assumption of added fluid volume in the work of Sinha [10] is thus only valid for the first mode of vibration, where the fluid added mass consists of a cylinder around the plate. The torsion modes situated within range of the first 5 modes seem to have nearly identical added mass factor values which can be at least 45%. This is the same as the conclusion reached by Haddara et Cao [3].

Looking at the asymptote of the added mass, these Figures show that the value of the added mass factor seems to be asymptotic at a D/L ratio of about 50%. But for rectangular plates (plate numbers 1, 3 and 4), convergence is earlier than for square plates (plate numbers 2 and 5). One can see that the acceptable D/L ratio where the added mass factor ceases to change for rectangular plates is about 0.3 to 0.4 and for square plates is between 0.4 and 0.5.

Observing the two CFFF plates numbered 1 and 3, the aspect ratio A/B of one plate is the inverse of the other (A/B=1.87 for plate 1, 0.53 for plate 3). The effect of change of added mass follows the same rule and depends on the longer dimension of the plate, whether this side is clamped or free.

5.4 Effect of damping change

Similar to frequencies, damping rates are identified at different depths and shown in the diagrams for plate number 4.

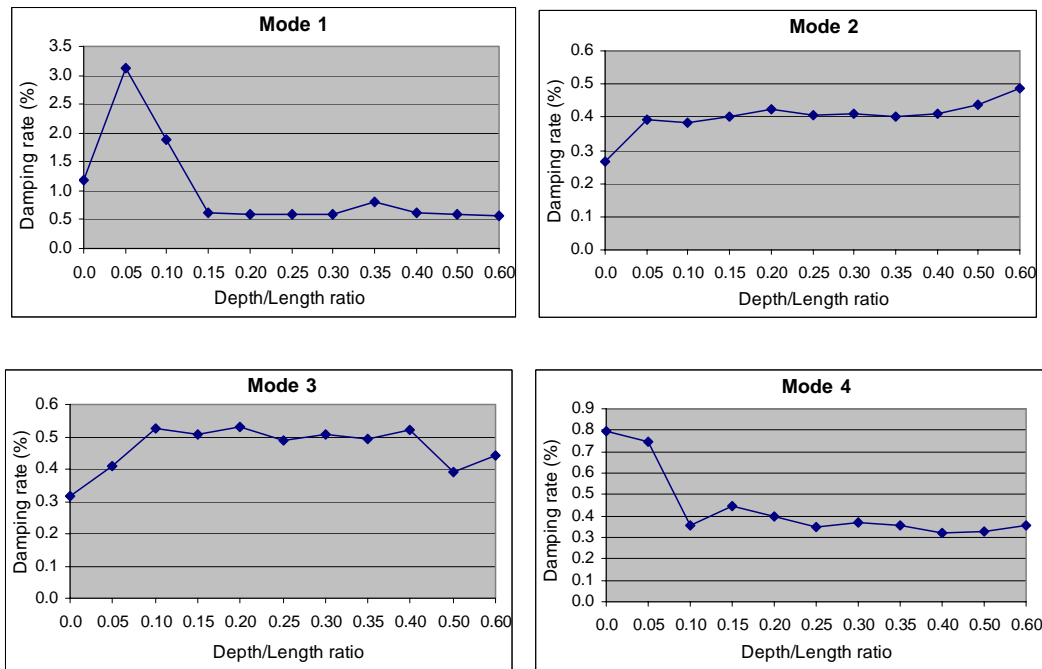


Fig. 15 Damping change effect on plate 4

Contrary to the frequencies, there is either a decrease (mode 1, mode 4) or increase (mode 2, mode 3) of damping when the plate is put into water. Since damping is difficult to identify and can be a source of discrepancy as discussed earlier, the damping ratio of a mode at different

depths would be considered constant if the fluid flow is not turbulent at the excitation time. This means that turbulence causes additional damping effects. It can also be seen that the first mode exhibits significant added damping value at low depth positions, which makes the total damping rate abruptly increase.

5.5 Mode shapes

The identified mode shapes are estimated using the Modal Assurance Criterion (MAC, see [9]) which describes the correlation between the identified vector and the analyzed vector of each eigen-mode. Figure 16 shows the MAC of the first 4 modes of plate 1.

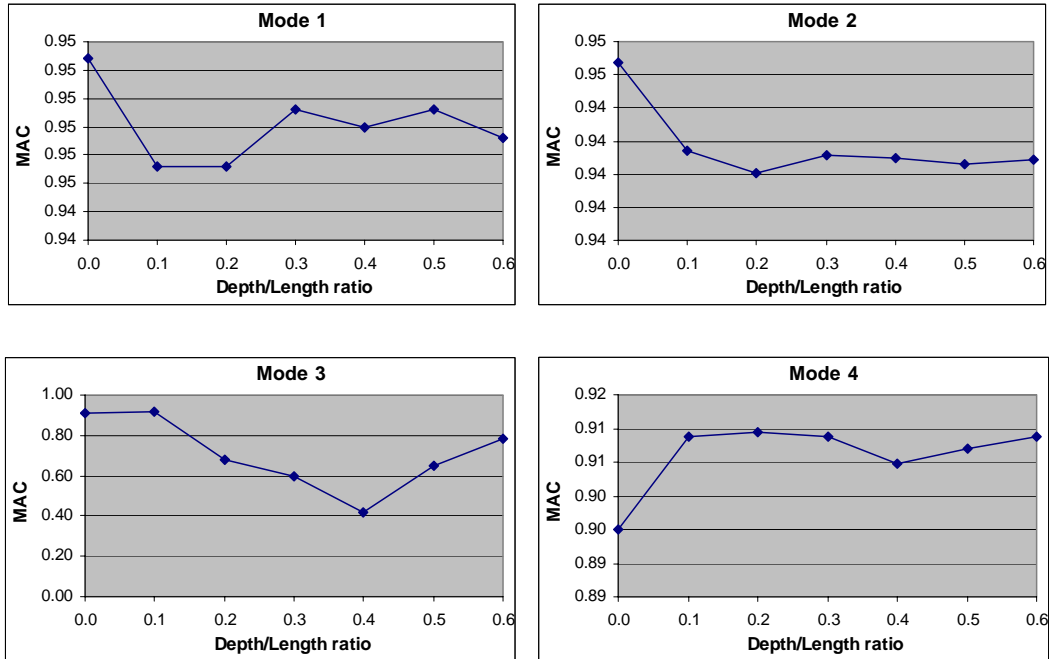


Fig. 16 Mode shapes of plate 1 (CFFFA377B201)

From these Figures one can see that the mode shapes are accurately identified when MAC values are close to 1. The mode shapes of the first ten modes are also unchanged when the structure moves from a vacuum into fluid. Only mode 3 is abnormal at several plate positions. This can be explained by poor modal excitation (low modal participation factor).

6. CONCLUSION

A relatively complete experiment on vibration of a plate structure submerged in water is presented in this paper, which complements fluid-structure interaction literature and validates certain analytically developed models. The effect of added mass on submerged plates and other types of structures is accurately derived using a fast time domain method. The following conclusions can be drawn from our observations and calculated results:

i) The added mass factor is distinctly observed when the plate is submerged in fluid. It changes sharply when the depth of the plate is less than one tenth of its length. As the plate continues lower into the fluid, the added mass factor changes slightly and converges at a specific depth, ranging from 30% to 40% for rectangular plates or 40% to 50% for square plates.

ii) The value of added mass is most significant for the first bending mode, which can be more than ten times the modal mass in air. This value is smaller for higher frequency bending modes but is nearly constant for all torsion modes. In the first five modes, the added mass factor attains at least 45% of the modal mass of the plate itself.

iii) The damping ratio of a mode is not constant. It can be higher when a noticeable excitation is applied. In almost all cases, damping increases when the plate is submerged, however another mode exists with a damping rate that is smaller than in air, although its value seems to tend towards convergence when the plate submerges deeper. The vibration of the plate can cause turbulence in the vicinity of the plate and the result is an abrupt increase in the total damping rate.

iv) Mode shapes are accurately identified, even when the plates are submerged in fluid. In the range of the first ten modes, there is a change or permutation of every mode shape.

v) Further studies are recommended to more accurately identify the critical D/L ratio where the frequencies start to change. The value of converged added mass for each mode should also be analytically derived. During these further tests and analyses, the effect of added or reduced damping could be carefully measured using a reasonable excitation force. A simple model for the prediction of added damping due to turbulence could be also developed.

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