A TIME DOMAIN METHOD FOR MODAL IDENTIFICATION OF VIBRATORY SYGNALS

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ABSTRACT

Vibration is one of the important aspects in risk engineering and security. In the U.S.A. alone there are in 1974 some 8-10 millions people who are regularly exposed each day to occupational vibration. Since the level of vibration exposure depends on the natural frequencies of the system, knowing the modal parameters of vibration human-related system is important to evaluate the vibration. This paper presents a method in the time domain, which can help experts to identify modal parameters of a system from the vibration responses measurement. A multivariate autoregressive model is introduced, in order to represent the dynamic response of the structure. The model parameters are estimated by the least squares method implemented via the QR decomposition technique. The derived method exposes a rapid, accurate procedure which can give out all dynamical parameters of the system and of excitation source.

RÉSUMÉ

La vibration est l'un des aspects importants dans la technologie et la sécurité de risque. Seuls aux Etats-Unis il y a dans 1974 environ 8-10 millions de personnes qui sont régulièrement exposés chaque jour à la vibration professionnelle. Puisque le niveau de l'exposition de vibration dépend des fréquences normales du système, savoir les paramètres modaux du système humain-connexe de vibration est important pour évaluer la vibration. Cet article présente une méthode dans le domaine temporel, qui peut aider des experts à identifier des paramètres modaux d'un système de la mesure de réponses de vibration. Un modèle auto-régressif multivariable est présenté, afin de représenter la réponse dynamique de la structure. Les paramètres modèles sont estimés par la méthode des moindres carrés appliquée par l'intermédiaire de la technique de décomposition de QR. La méthode dérivée expose un procédé rapide et précis qui peut donner dehors tous les paramètres dynamiques du système et de source d'excitation.

1. INTRODUCTION

Vibration can cause risks and daily vibration exposure can seriously and irreversibly hurt people who are exposed. In the U.S.A. alone there are in 1974 some 8-10 million people who are regularly exposed each day to occupational vibration and many more world-wide [1]. This number must be much more increased in recent years. The vibration exposures as described in [2, 3] can be classified in to two groups depending on the job.

- The first class is called Whole-Body Vibration (WBV) which affects a head-to-toe exposure. One can example truck, heavy equipment, railroad, etc.
- The second type is called Hand-Arm Vibration (HAV) or localized exposure. This class includes all type of pneumatic, electrical and hydraulic hand-tools.

It can be seen that the motion in vibration is complicate. Since it relates to the human security, there are numerous occupational vibration standards used worldwide for the WBV and HAV. For example in international scale there are ISO 5394 for HAV, ISO 2631-1997 for WBV. In U.S, they also have ANSI S3.34 for HAV and ANSI S3.18 for WBV. The common conceptual these standards on the measurement and evaluation of vibration with respect to human response is illustrated on the Figure 1 [3].

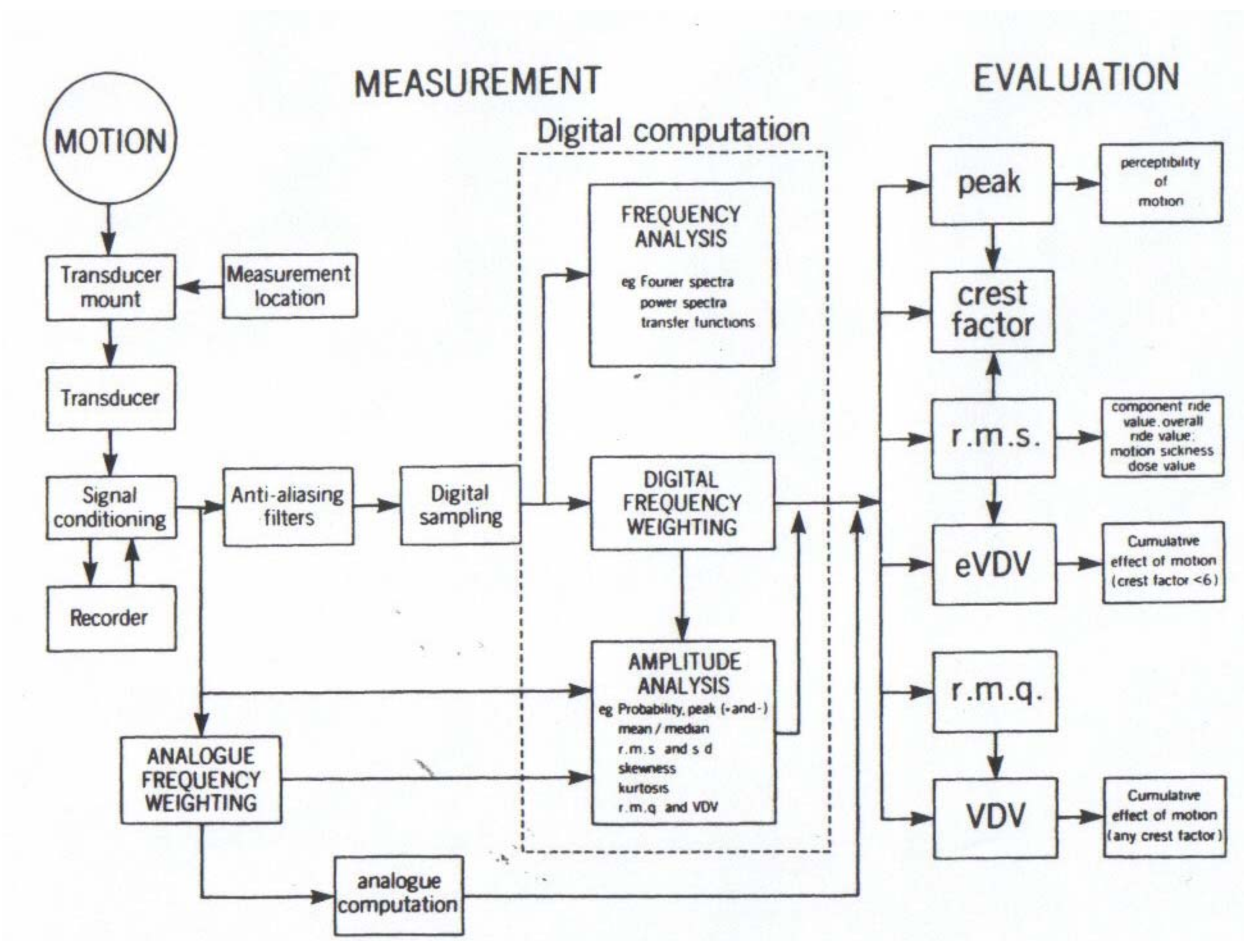

Fig. 1 Conceptual of measurement and evaluation of vibration with respect to human

(*Source: M. Griffin, 1990, "Handbook of Human Vibration", Academic Press London- [3]*)

The dynamic responses of mechanical system, including the human body, are dependent on the frequency of vibration which has to be cautiously analysed in the digital computation. Since the vibration exhibits all its behaviour in the modal characteristics, it is most suitable to examine the “resonance” or “natural frequency”. Research has showed that the human body is very sensible to several ranges of resonant frequency wherein human are most vulnerable over others. ISO 2631 designates a human WBV resonance occurs at in the vertical direction from 4-8Hz, in the side-to-side and front-to-rear at 1-2Hz [2].

In the existing codes for evaluation of human exposure to whole-body vibration ISO2631 (1997) [4], natural frequencies are important indices to:

- Get the acceleration limits for each frequency when evaluating the ‘‘Fatigue Decreased Proficiency- FDP’’, the ‘‘Reduced Comfort- RC’’ or ‘‘Exposure limits- EL’’ to see if one of these desirable limits has been exceeded for the respective exposure time.
- Get the acceleration frequency weighting for each vibratory axis in order to evaluate all human responses.

It can be seen in the codes that, outside of the above sensitive range (4-8Hz for z axis, 1-2Hz for x and y axis); these parameters are highly dependent of the exactitude of the frequency identification as shown in Figure 2.

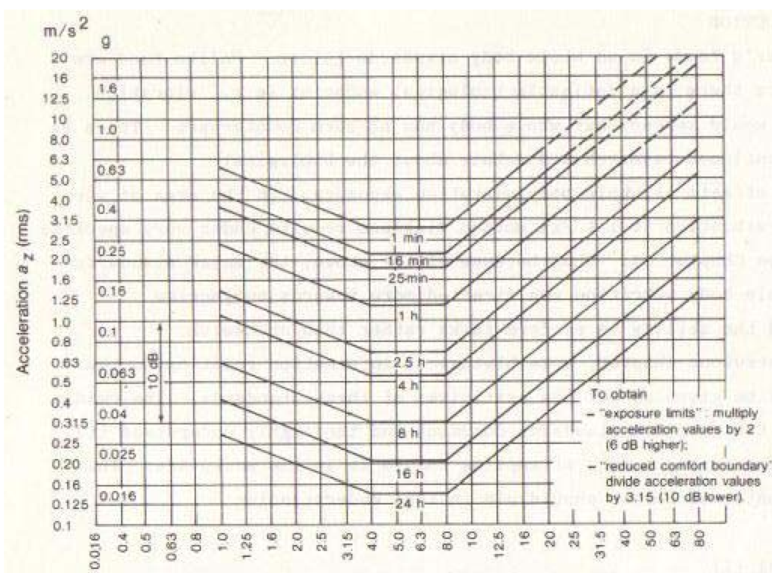

Fig. 2 Acceleration limits as a function of vibration frequency and exposure time

(*Source: Extracted from ISO2631-1997 [4]*)

For the identification of the natural frequencies of a structure or system, “*modal testing*” technique has been applied and be conducted in the frequency domain as the benchmark method, referred in the code ISO7626-2 (1990) [5]. The classical vibration analysis deals with the using of analogue filters such as Octave or one-third octave band, but these methods fail to provide a detailed profile of vibration exposure [3]. Modern frequency analysis has conducted in the frequency domain and uses the Fourier transform to calculate the modal parameters by the peak-picking method. But one problem with using FFT for human analysis is that the greater the duration of measurement that is to be analyzed, the narrower the frequency resolution is obtained [6]. Furthermore, several difficulties can exhibit in the decision of structural modes or harmonics, closely modes and very light or high damping level [7]. One can found in vibration analysis for the assessment of WBV complied with ISO 2631 a numerous papers working with the method of frequency domain, such as Griffin [8], Cann [9] and Valsickle [10].

In the recent years, time domain methods have dealt in order to provide a more accurate identification technique which is faster and more efficient. One can cite several well known methods for the identification of a free vibration temporal response data, such as Ibrahim time domain method (ITD) [11], Least squares complex exponential (LSCE) [12], etc. Recently, Rutzel [13] has developed a modal description for the analysis of Whole body vibration through the illustration of the apparent mass. The modal parameters are identified by the minimization of the error function expressed in term of that apparent mass error.

It has been obvious seen that the temporal response of a system is a dynamical process. Then a time series model can be applied to represent the data. Among time series models, Autoregressive based model is most suitable for a linear system with time invariant parameters. This model has been widely applied in structural and mechanical analysis, for example [14, 15].

2. DEVELOPMENT OF THE TECHNIQUE

2.1 Multivariable autoregressive model

The autoregressive model for multivariate of order p ARV (p) can generally be expressed in the form [7].

$$y(t) + A_1 y(t-1) + A_2 y(t-2) + \ldots + A_p y(t-p) = w(t) \tag{1}$$

where $y(t-i)\big|_0^p\big|_{dx1}$ is the delayed output vector of the time interval $i.k\Delta t$ and k is normally unity, $A_i\big|_1^p\big|_{dxd}$ is the autoregressive parameter matrix, relating the output $y(t)$ to $y(t-i)$, and $w(t)\big|_{dx1}$ is the residual vector of all output channels, considered as the error of the model.

Equation (1) can be rewritten in the form of a regression:

$$y(t) = \Phi^T \varphi(t) + w(t) \tag{2}$$

Here $\Phi\big|_{dpxd} = \left[-A_1 \quad -A_2 \quad \ldots \quad -A_p\right]^T$ regroups all parameters matrices whose i^{th} column consists of the coefficients associated with the i^{th} component of $y(t)$ and $\varphi(t)\big|_{dpx1} = \left[y(t-1); \quad y(t-2); \quad \ldots; \quad y(t-p)\right]$ is used as the same set of regressors for each component of $y(t)$.

If we consider N consecutive values of the responses from $y(k)$ to $y(k+N-1)$, the model parameters can be obviously estimated by least squares, i.e., meaning the minimizing the sum of squares of the residual [16].

$$\hat{\Phi} = \arg\min V_N(\Phi) = \arg\min\left(\frac{1}{N}\sum_{t=k}^{k+N-1}\left\|w(t)\right\|^2\right) = \arg\min\left(\frac{1}{N}\sum_{t=k}^{k+N-1}\left\|y(t) - \Phi^T\varphi(t)\right\|^2\right) \tag{3}$$

In equation (3) arg min means “the minimizing argument of the function” and $V_N(\theta)$ is a welldefined scalar valued function of the model parameters. The vector of parameters estimated from equation (3) is called the least squares estimate.

2.2 Model parameters estimation

The data of these N consecutive values of the responses can be cast in term of moment matrices

$$U\Big|_{dpxdp} = \sum_{t=k}^{k+N-1} \varphi(t).\varphi^T(t)\,; \qquad V\Big|_{dxd} = \sum_{t=k}^{k+N-1} y(t).y^T(t)\,; \qquad W\Big|_{dxdp} = \sum_{t=k}^{k+N-1} y(t).\varphi^T(t)\,. \tag{4}$$

The estimated parameters matrix is therefore simply derived and can be written as [17], referring to the ordinary least squares method:

$$\hat{\Phi}\Big|_{dxdp} = W.U^{-1} \tag{5}$$

In equation (5), it is seen that the matrix $U\Big|_{dpxdp}$ has very high dimension and may be ill-conditioned. This technique is hence avoided in case of multivariable model. The following section describes an efficient way using QR-factorization. Reader can refer to Golub and Van Loan [18] for a thorough description of the QR-factorization.

If one form the data matrix

$$K\Big|_{Nx(dp+d)} = \begin{bmatrix} \varphi^T(k) & y^T(k) \\ \varphi^T(k+1) & y^T(k+1) \\ \ldots & \ldots \\ \varphi^T(k+N-1) & y^T(k+N-1) \end{bmatrix} \tag{6}$$

The moment matrices can be cast in

$$M\Big|_{(dp+d)x(dp+d)} = \sum_{t=k}^{k+N-1} \begin{bmatrix} \varphi(t) \\ y(t) \end{bmatrix} \begin{bmatrix} \varphi^T(t) & y^T(t) \end{bmatrix} = \begin{bmatrix} U & W^T \\ W & V \end{bmatrix} = K^T.K \tag{7}$$

Introducing now the QR factorization of the data matrix, that means the matrix is decomposed as

$$K = Q.R \tag{8}$$

With $Q\Big|_{NxN}$ is an orthogonal matrix (that is $Q.Q^T = I$) and $R\Big|_{Nx(dp+d)}$ is an upper triangular matrix

$$R = \begin{bmatrix} R_1\Big|_{dpxdp} & R_2\Big|_{dpxd} \\ 0 & R_3\Big|_{dxd} \\ 0 & 0 \end{bmatrix} \tag{9}$$

This decomposition gives the moment matrix in a new form

$$M = (Q.R)^T.(Q.R) = R^T.R = \begin{bmatrix} R_1^T R_1 & R_1^T R_2 \\ R_2^T R_1 & R_2^T R_2 + R_3^T R_3 \end{bmatrix} \tag{10}$$

From equation (5), with attention to the uniformity of (7) and (10), the parameters matrix is then rewritten in the new form

$$\hat{\Phi} = (R_2^T R_1).(R_1^T R_1)^{-1} = (R_1^{-1} R_2)^T \tag{11}$$

2.3 Modal parameters extraction

Once the model parameters are estimated, the state matrix of the system can be established as in form of autoregressive parameters [7]

$$A\Big|_{(dpxdp)} = \begin{bmatrix} -A_1 & -A_2 & -A_3 & \dots & -A_p \\ I & 0 & 0 & \dots & 0 \\ 0 & I & 0 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & I & 0 \end{bmatrix} \tag{12}$$

The poles of model are also the roots of characteristic polynomial of the state matrix. Therefore one can get from the eigenvalue problem.

$$[V, \lambda] = eig(A) \tag{13}$$

The complex eigenvalues λ give the frequencies and damping rates of the system.

Angular frequency: $$\omega_i = \sqrt{\mathrm{Re}^2(\lambda_i) + \mathrm{Im}^2(\lambda_i)} \tag{14}$$

Damping ratios: $$\xi_i = -\frac{\mathrm{Re}(\lambda_{i)}}{\omega_i} \tag{15}$$

It is seen that the number of rows of matrix V is dxp. The eigenvector of the mode shape is taken from the first d values in each column of matrix V. Since the mode shape can also analyzed by Finite element method, the term “MAC- Modal assurance criterion” defines the correlation coefficient of this analyzed eigenvector value and the identified one.

3. VALIDATION ON SIMULATED DATA

3.1 Simulation system

A system of 2 degrees of freedom is considered to simulate in order to provide vibratory temporal response data. The system consists of two lumped masses on an infinite rigid beam which vibrates on a system of springs and dampers (Figure 3). In order to identify also the harmonic excitation, a sinusoidal force is applied on the second mass. The frequency of the excitation can be chosen equal or unequal to the resonances of the system itself.

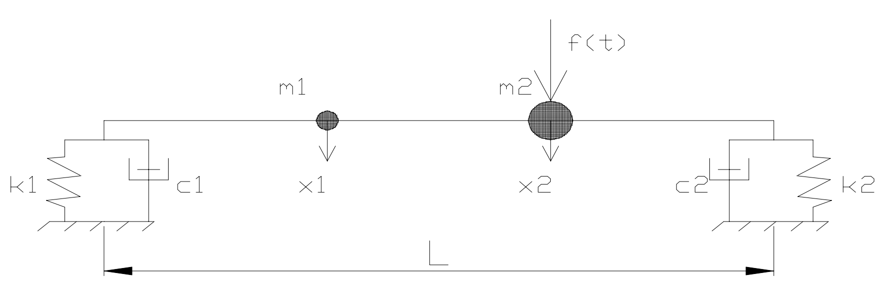

Fig. 3 System 2DOF

The governing equation of the dynamics of the system is:

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix}\begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{bmatrix}+\begin{bmatrix} 4c_1+c_2 & -2c_1-2c_2 \\ -2c_1-2c_2 & c_1+4c_2 \end{bmatrix}\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix}+\begin{bmatrix} 4k_1+k_2 & -2k_1-2k_2 \\ -2k_1-2k_2 & k_1+4k_2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}=\begin{bmatrix} 0 \\ f(t) \end{bmatrix} \quad (16)$$

For the convenience, the physical model of system is given first in form of matrices

$m_1 = 100kg$, $m_2 = 200kg$; $c_1 = c_2 = 500Ns/m$; $k_1 = 10^6 N/m$, $k_2 = 2.10^6 N/m$

$$[M]=\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix}=\begin{bmatrix} 100 & 0 \\ 0 & 200 \end{bmatrix}(kg); \quad [C]=\begin{bmatrix} 2500 & -2000 \\ -2000 & 2500 \end{bmatrix}(Ns/m); \quad [K]=\begin{bmatrix} 6.10^6 & -6.10^6 \\ -6.10^6 & 9.10^6 \end{bmatrix}(N/m)$$

The two natural frequencies, damping ratios and mode shapes are easily to be calculated, giving:

$$f=\begin{bmatrix} f_1 \\ f_2 \end{bmatrix}=\begin{bmatrix} 15.4448 \\ 49.2016 \end{bmatrix}Hz; \quad X_1=\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}_1=\begin{bmatrix} 1.000 \\ 0.843 \end{bmatrix}; \quad X_2=\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}_2=\begin{bmatrix} 1.000 \\ -0.592 \end{bmatrix}; \quad \xi_2=\begin{bmatrix} \xi_1 \\ \xi_2 \end{bmatrix}=\begin{bmatrix} 1.9249\% \\ 5.4609\% \end{bmatrix}$$

It is discussed in the previous work that method can give an accurate identification in the noise to signal level up to 60dB. In this research, a random noise of 40dB is generated in all output channels, gives a sample of response as shown in Figure 4 at sampling frequency of 200Hz.

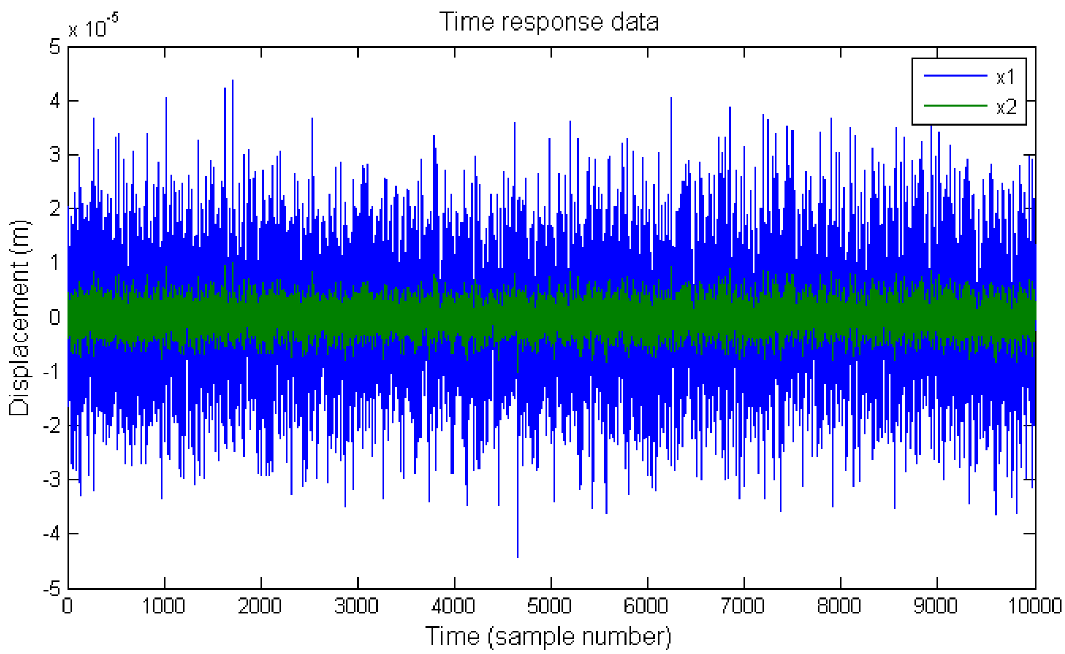

Fig. 4 Simulated data of 2DOF system

3.2 Modal parameter identification

Equation (13) shows the existence of a numerous eigenvalues of the system with a given order. It means the modal parameters would be identified based on its stability within a range of model orders. This technique of representation has also an advantage for the users to interact, choose and decide the structural modes out of spurious modes. In the previous investigation [14], it is seen that once the necessary order is attempt, the structural frequency will be stable with negligible variance. The maximum order to be chosen need not to be the optimal order as required in several criterions but to be enough to attain stability in damping identification. As reported in that research, this maximum order is only depended on the noise level and can be appropriately chosen from 30 to 50. Natural frequencies and so on damping rates are taken the median values of the stable values vector from the necessary to the maximum order. Figure 5 to Figure 8 and Table 1 show the result of the system with the model order up to 50.

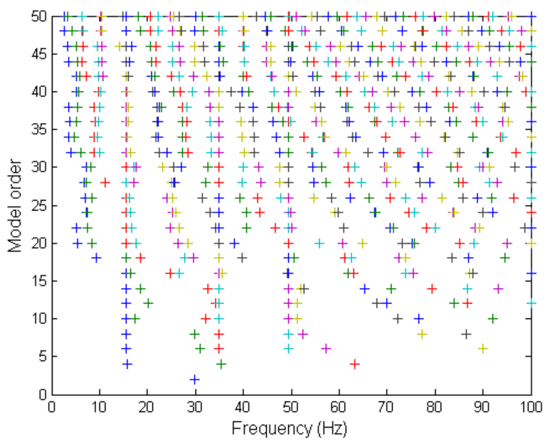

Fig. 5 Frequency stability

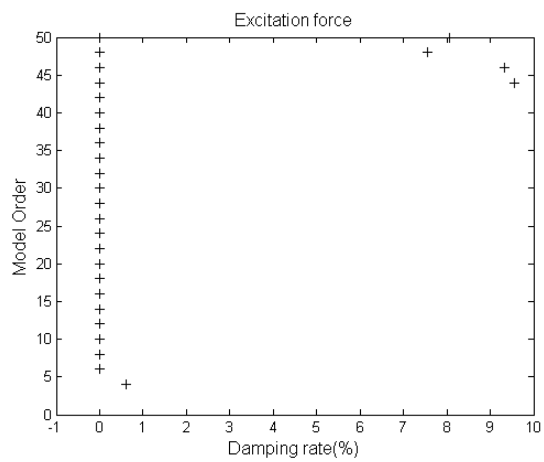

Fig. 6 Damping stability of excitation

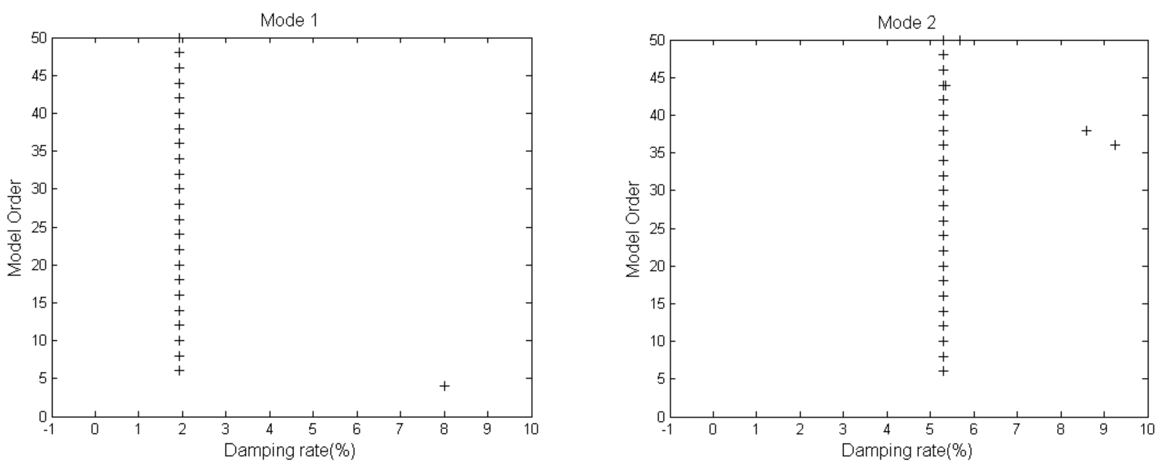

Fig. 7 Damping stability of structural modes

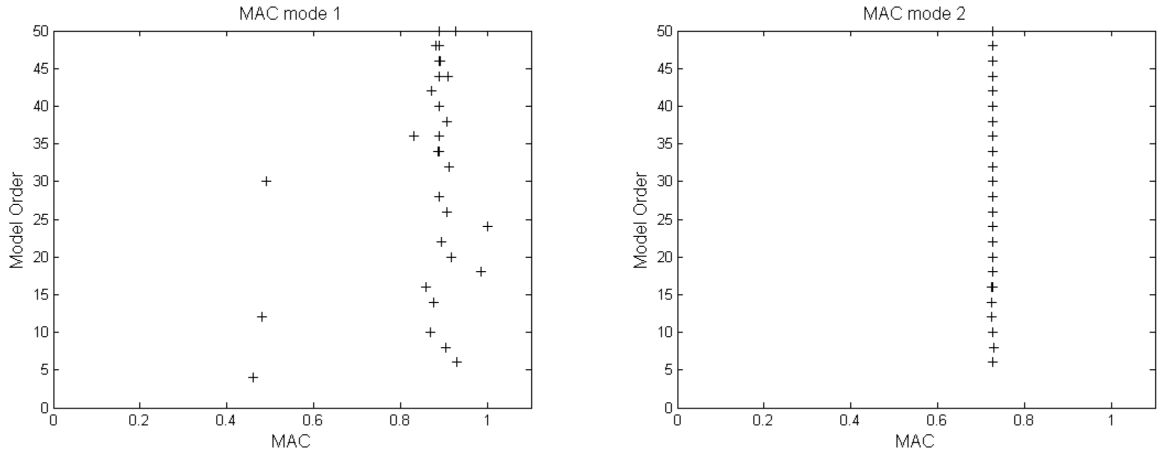

Fig. 8 Mode shape stability of structural modes

Tab. 1 Identification result of 2DOF system

Mode	Frequency (Hz)			Damping rate			MAC		
	Simulated	Identified	Error (%)	Simulated	Identified	Error (%)	Simulated	Identified	Error (%)
1	15.44	15.4449	0.0	1.92	1.9255	0.3	1.00	0.8901	11.0
2	49.20	49.3998	0.4	5.46	5.2942	3.0	1.00	0.7253	27.5
Excitation	35.00	35.0000	0.0	0.00	0.0000	0.0			

It is seen that the method provides an accurate identification of system, even in case of high noise level. A perfect coherence of frequencies is attempt and a negligible error of damping rates and mode shapes is observed. The presence of harmonic excitation is insured by the vanished value of its corresponding damping rate which is more effective than the frequency domain method.

4. APPLICATION ON DYNAMIC TESTING

In this application, we consider a case of Whole Body Vibration (WBV) in taking a dynamic testing on a steel beam. This structure is very commonly used in industry such as a fundamental component. It consists of a U shape steel simple supported beam whose characteristics are given in Table 2. The test configuration is given in Figure 9 with three accelerations attached on the beam. Data is acquired and sampled at frequency of 1280Hz through the ZONIC card with the companion software eZ-Analyst Medallion [19].

Girder length	142.5cm
Cross section	3.33cm2
Inertia moment	2.99cm4
Elastic modulus	2e11 Pa
Poisson coefficient	0.29
Self weight	7850 kg/m^3

Tab. 2 Beam properties

Fig. 9 Configuration of tested beam

4.1 Free vibration of the beam

The beam is first excited by an impulsion on the structure itself. An impact hammer is used to attack at the mid-span of the beam, give a response in Figure 10. Figure 11 to Figure 13 and Table 3 show the result of identification.

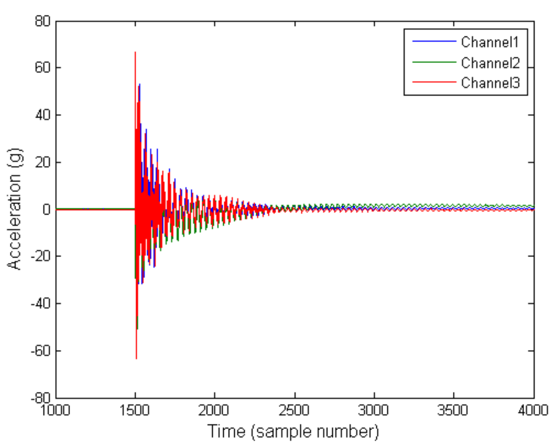

Fig. 10 Free vibration data of beam

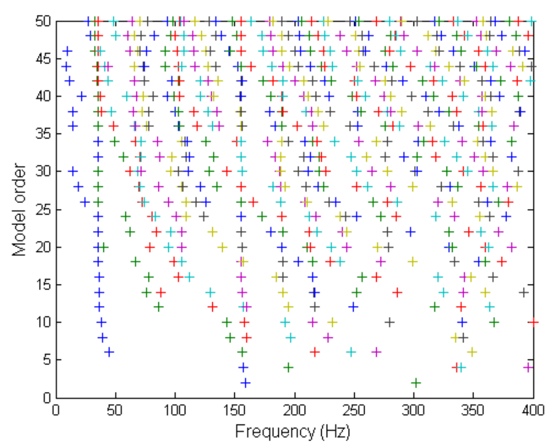

Fig. 11 Frequency stability of beam

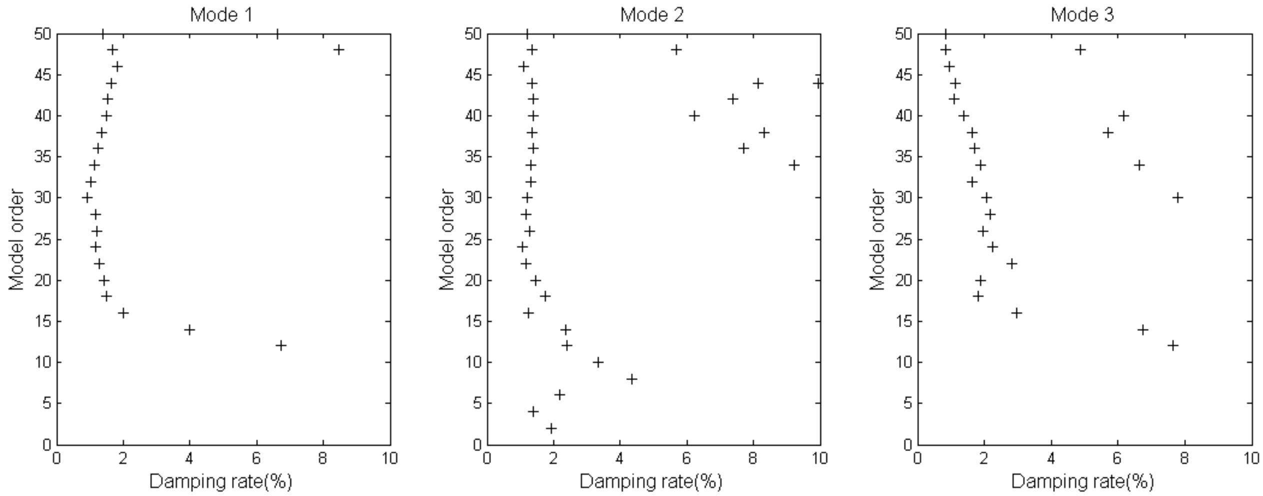

Fig. 12 Damping stability of beam

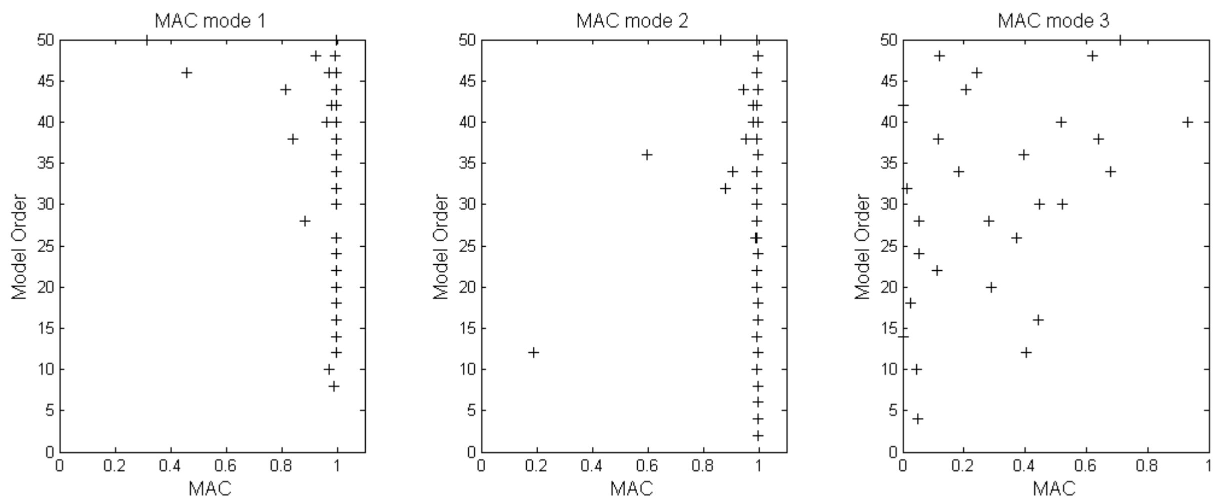

Fig. 13 Mode shapes stability of beam

Tab. 3 Identification result of beam

Mode	Frequency (Hz)			Damping rate	MAC		
	Simulated	Identified	Error (%)	Identified	Simulated	Identified	Error (%)
1	37.00	35.386	4.4	1.4709	1.00	0.9967	0.3
2	148.01	155.4722	5.0	1.3626	1.00	0.9951	0.5
3	333.03	189.7507	43.0	1.8802	1.00	0.2619	73.8

The free vibration of the beam is accurately identified with the first two modes in the frequency range. The error of frequency identification is less than 5%. Several stable frequencies can be existed on the diagram but they do not refer to any structural mode therefore the error of its corresponding frequency and MAC is unacceptable.

4.2 Free vibration of the beam and added mass

A dead motor is disposed at the mid span of the beam. The machine is considered as an added mass to the beam structure. Results are shown on Figure 14 to 17 and Table 4.

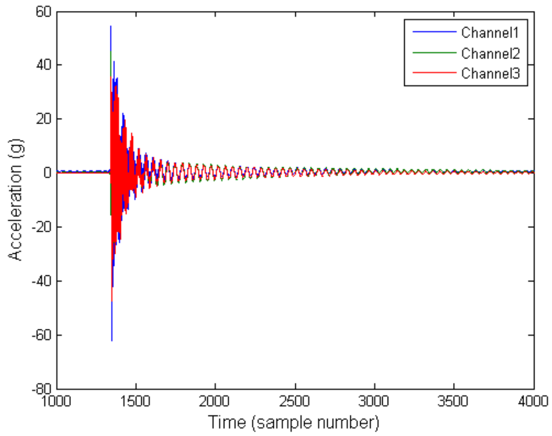

Fig. 14 Free vibration data with rotor

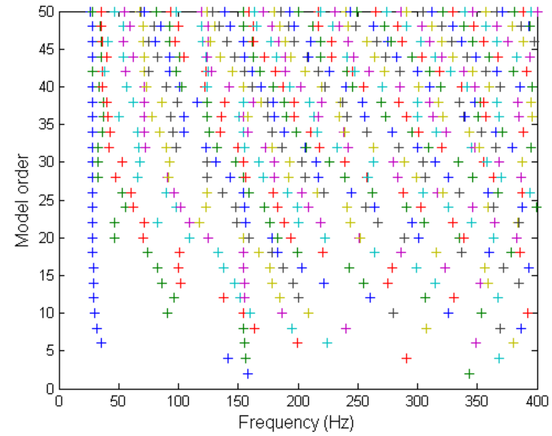

Fig. 15 Frequency stability with rotor

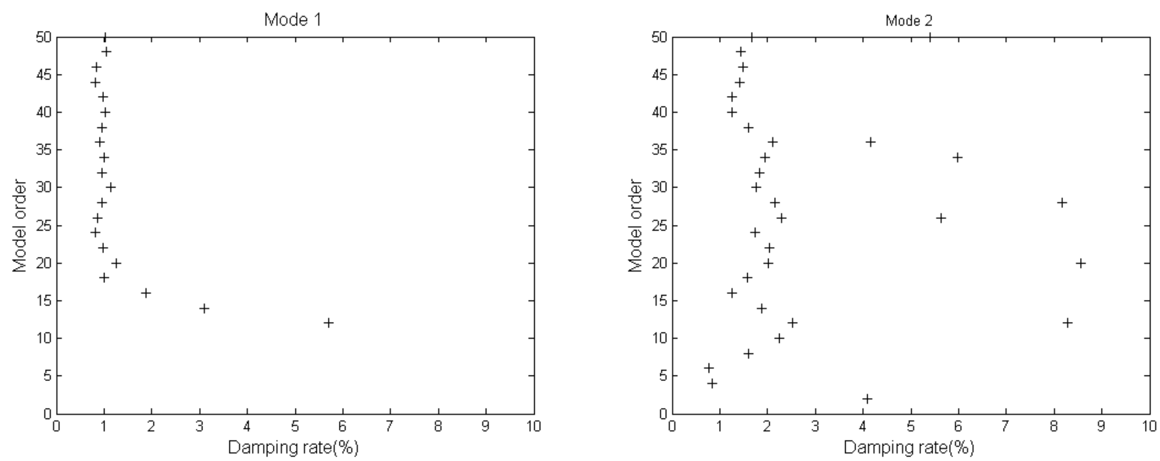

Fig. 16 Damping stability with rotor

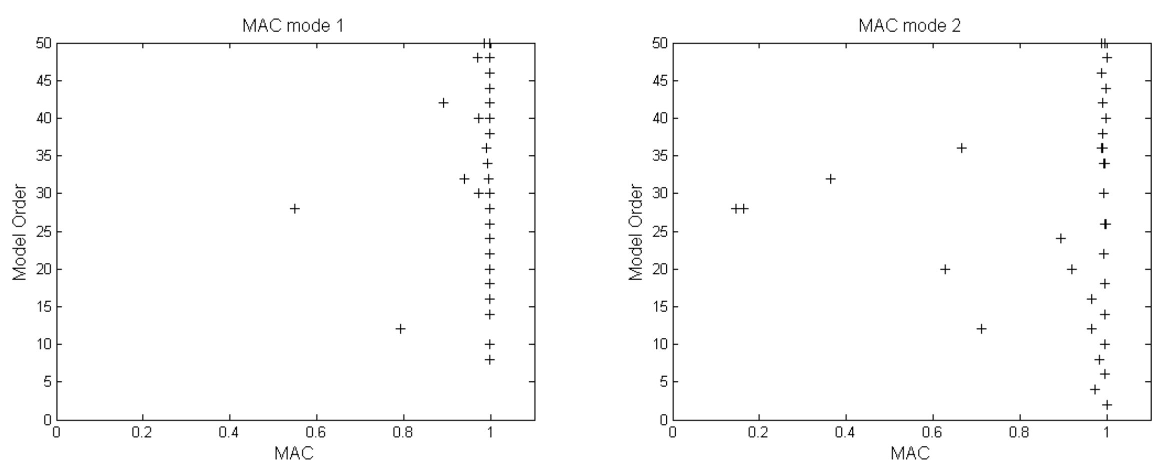

Fig. 17 Mode shape stability with rotor

Tab. 4 Identification result with rotor

Mode	Frequency (Hz)			Damping rate	MAC		
	Simulated	Identified	Error (%)	Identified	Simulated	Identified	Error (%)
1	27.15	28.0590	3.3	1.0121	1.00	0.9968	0.3
2	148.21	154.6375	4.3	1.7493	1.00	0.9894	1.1

With the added mass value due to the presence of human or equipments, the natural frequencies of the structure decrease. The change of frequency depends of the value added mass and its position and is significant for the first bending mode. There is a slightly change of damping before and after the presence of added masses but this value can not insure a real change of damping property of the structure.

4.3 Forced vibration of the beam by a rotor

The rotor is now running in order to excite the force vibration of the beam. The excited frequency of the rotor is constant at 120Hz. Results are shown at same fashion on Figure 18 to 22 and Table 4.

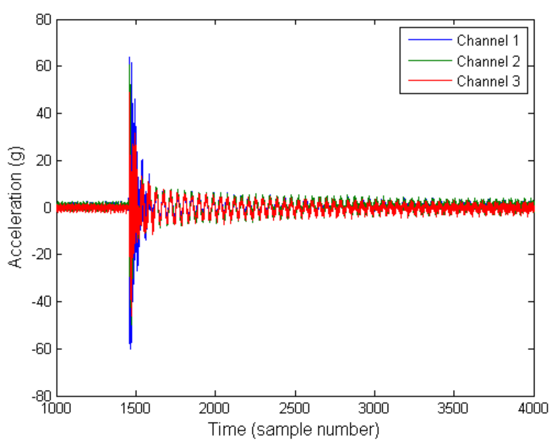

Fig. 18 Forced vibration data

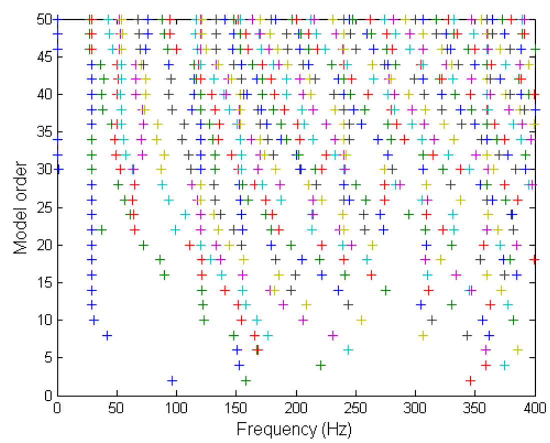

Fig. 19 Frequency stability of forced data

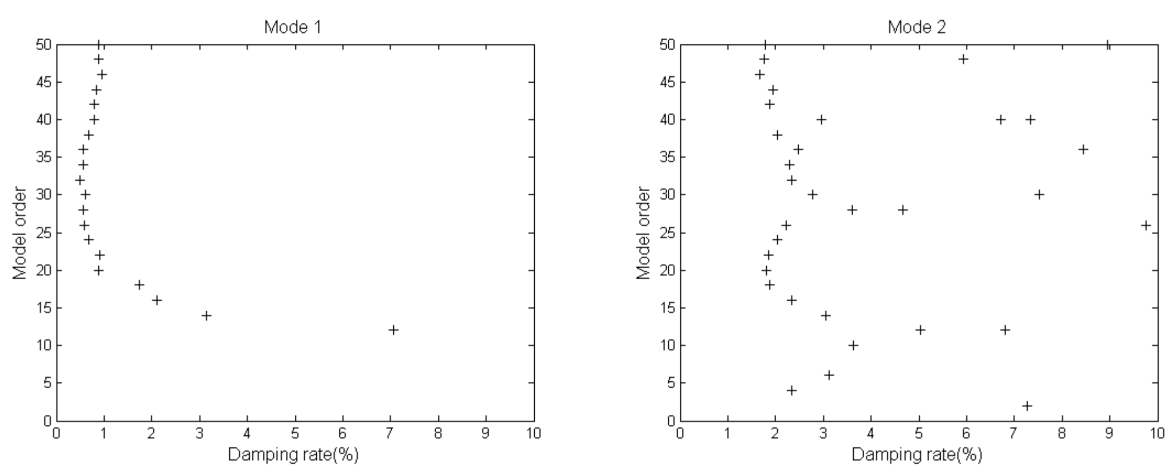

Fig. 20 Damping stability of forced data

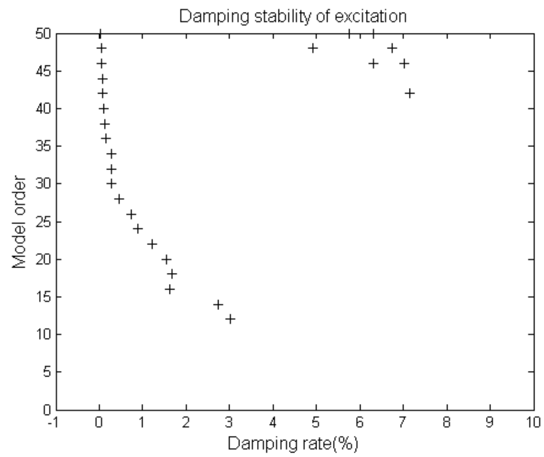

Fig. 21 Identification of excitation

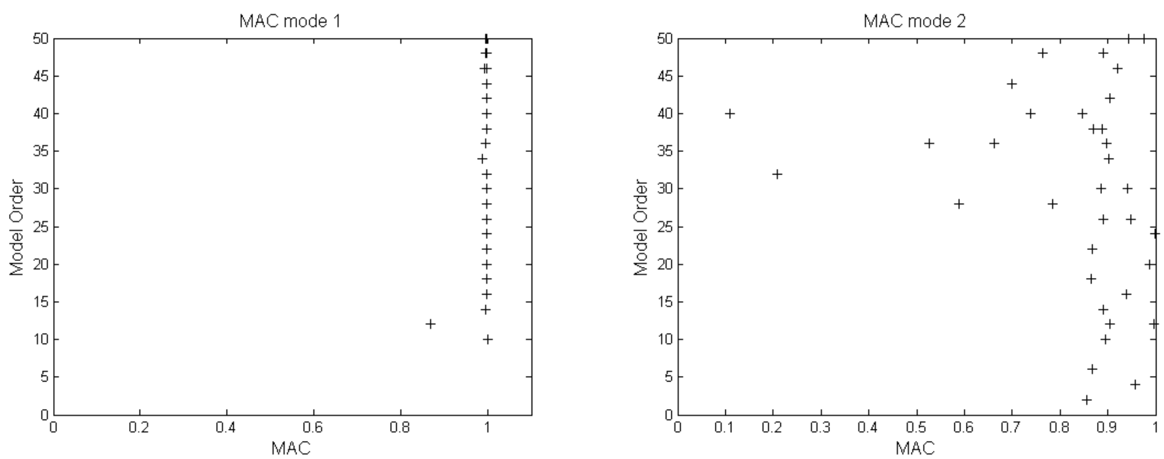

Fig. 22 Mode shape stability of forced data

Tab. 5 Identification result of forced vibration data

Mode	Frequency (Hz)			Damping rate	MAC		
	Simulated	Identified	Error (%)	Identified	Simulated	Identified	Error (%)
1	27.15	28.7344	5.8	0.8296	1.00	0.9974	0.3
2	148.21	153.8597	3.8	2.3142	1.00	0.8895	11.1

On the frequency stability diagram, one can see a numerous stable frequencies. In the frequency domain, if we have not a priori knowledge about the excitation, it is difficult to distinguish the spurious mode to the structural one. This method, with the superiority of using the stability diagrams, can give a better decision on the harmonic components. It is seen also that with the presence of forced vibration, the damping rates of the structural modes can be always accurately identified.

5. CONCLUSIONS

A method for modal identification of vibrating structures is presented in the time domain. The implementation of the least squares by the using of QR factorisation gives a fast and more conditioned algorithm. With the familiar representation in the stability diagrams of the modal parameters, this method provides an effective decision and distinguishing of structural modes from the spurious ones, therefore the modal parameters are accurately identified, even in cases of very high random noise level. This method is thus can be developed for the vibration analysis of all kind of structures, especially in industrial security where the whole body vibration exposure caused by the structures, system or machines is vulnerable and important. Further validation of the method would be considered in case of identification of vary-frequency excitation and of a biodynamic system where the nonlinear vibration signal from human bodies has to be taken into account. Authors suggest also an example demonstration on the derivation of the level of exposure indications such as the root-mean-square, peak acceleration, crest factor or vibration dose value (VDV) from this time domain method.

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