ROTOR HEALTH MONITORING BY MODAL ANALYSIS

SURVEILLANCE DES FISSURES DE ROTORS PAR ANALYSE MODALE

Thomas Marc.[1] *Lakis Aouni.A.*[2] *Hamidi L.*[3] *Massoud M.*[3]
marc.thomas@etsmtl.ca
[1] Department of Mechanical Engineering, École de technologie supérieure, Montreal (Qc), Canada, H3C1K3
[2] Department of Mechanical Engineering, École Polytechnique, Montreal (Qc), Canada.
[3] Department of Mechanical Engineering, Université de Sherbrooke (Qc), Canada

ABSTRACT
Mathematical models of cracked rotor systems are used to study the effect of the presence of open cracks on the system's modal parameters. The system's parameters are expressed in terms of dimensionless quantities and the performance is characterised by a natural frequency index and an elastic line index. The effects of crack depth, crack location, rotational speed and slenderness of the rotor on the natural frequency and the elastic line indexes is investigated for three rotor configurations. It is shown that the natural frequency index may be used to determine the depth of the crack and the elastic line index locates it.

RESUME
Les modèles mathématiques d'ensembles-rotor fissurés sont étudiés pour en extraire les effets produits par la présence d'une fissure ouverte sur les paramètres modaux du système. Les paramètres sont exprimés en termes de quantités sans dimensions et la performance du modèle est caractérisée par un index de fréquences naturelles et un index modal. L'impact de la profondeur, de la position de la fissure, ainsi que de la vitesse de rotation et de la minceur (ou rapport longueur-diamètre) du rotor sur les indices de fréquences naturelle et de ligne élastique font l'objet d'une étude pour trois configurations de rotor. Nous démontrons que l' index fréquentiel peut être utilisé pour déterminer la profondeur de fissure et que l'index modal nous indique la position du défaut.

1. INTRODUCTION
In predictive maintenance, not only machinery and mechanisms defaults must be detected at early stage, but also structural damages, especially in welded joints[1], offshore platform [2] and bridges[3, 4, 5]. When a crack is suspected, the engineer may use many techniques in order to examine the crack, such as Foucault courant, eddy current, acoustical emission or ultra-sound. Previous steps are needed before inspection in order to detect the occurring of cracks. Vibration measurement is an easy and effective tool to implement [6, 7]. The aim of this study is to investigate effective monitoring techniques for detecting structural damage when it occurs [8, 9, 10]. Theoretically a crack produces a local decreases of the stiffness that induces a decrease in the natural frequency [11, 12, 13]. Two crack models may be considered [14]: the open crack [15] that is usually investigated by modal analysis [16 to 20] and the breathing crack [21, 22] or fatigue crack that produces non linear vibrations [23] and requires time-frequency analysis [24, 25]. Most of the studies investigate the effect of cracks in structures, but few have studied the effect of a crack on a rotor [26, 27]. This study is aimed to investigate the modal parameters behavior of an open cracked rotor. The crack model use a narrow slot in order to modelize an open crack. The mathematical model to describe the dynamics of the cracked rotor [28, 29, 30] was developed using a finite element

analysis. This model was found adequate and compared well with the exact solutions of simple configurations and with the experimental results [31]. Two indices of performance are used to compare the dynamics of a cracked rotor with those of an uncracked one. The first index is referred to as "the natural frequency index". The second is a newly introduced index and is referred to as "the elastic line index". This paper reports on a systematic parametric study of the open cracked rotor using the two indices with the aim of identifying the crack's location and its depth. The study is confined to the first mode of the rotor system since it is the most significant in the crack identification [32].

2. NATURAL FREQUENCY AND MODE VARIATION OF A STRUCTURE WITH AN OPEN CRACK

2.1 Dimensionless parameters

The rotor systems considered in this investigation are composed of a number of shafts in bending [33] which carry discrete disks and are supported by bearings at specific locations. This study assumes that only one open crack exists at a specific location. Each open cracked rotor is characterised by five dimensionless parameters, namely, M_i , R_i , S , X_i and Λ_i , which represent, respectively, the mass, radius of the i'th disk, rotor slenderness, disk location and its rotation. These parameters may be expressed as follows:

$$M_i = m_i / m; \quad R_i = r_i / r; \quad S = r / L; \quad X_i = x_i / L; \quad \Lambda_i = \frac{\Omega_i}{\Omega_c} \qquad \textbf{(1)}$$

where

L is the length of rotor system; m and r are the mass and radius of rotor shaft

m_i, r_i are the mass and radius of i'th disk; x_i is the location of the i'th disk;

Ω_i is the angular velocity of rotation of rotor system; Ω_c is the critical speed

Figure (1) shows the three basic rotor system configurations investigated.

- ✓ Configuration A considers a simply supported shaft.
- ✓ Configuration B considers a simply supported shaft carrying one disk located at $X_1 = 0.4$
- ✓ Configuration C considers a cantilever shaft carrying one disk at its extremity.

The location (μ_c) and depth of the crack (Ac) are denoted by the following dimensionless quantities:

$$\mu_c = \mu / L; \quad A_c = a / r \qquad \textbf{(2)}$$

where μ is the actual crack location on shaft and a is the crack depth (figure 2).

Two rotations Λ_1 and Λ_2 are selected to drive the rotor systems at speeds lower and higher than the critical speed. These speeds are given in Table 1 for each investigated configuration.

The parametric studies the above mentioned parameters (M_i, R_i, S, X_i, Λ_i, and A_c) that are all held fixed at some reference values, while μ_c is varied over a given range. The effects of this imposed variation on the two performance indices ε_e and ε_{f-} are recorded and plotted.

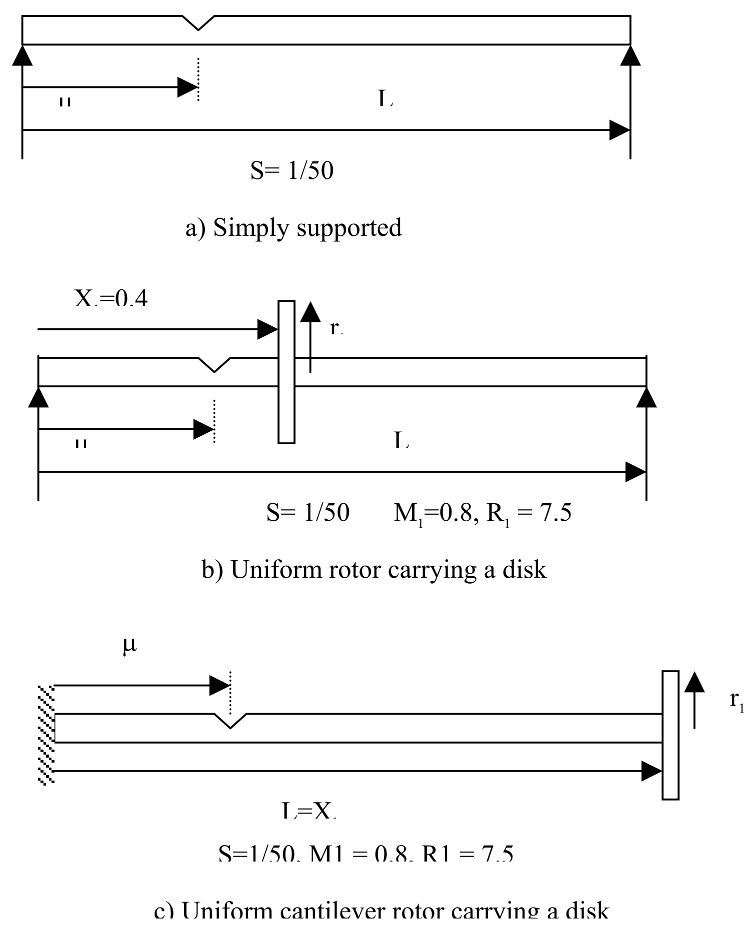

Figure 1 The rotor configurations

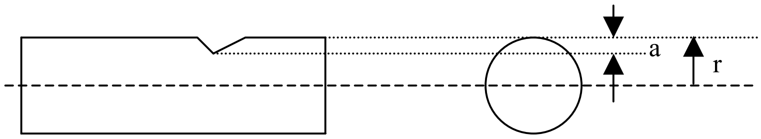

Figure 2 Crack depth

Table 1: Rotational parameters

Case	Ω_c (RPM)	$\Lambda_1 = \frac{\Omega_1}{\Omega_c}$	$\Lambda_2 = \frac{\Omega_2}{\Omega_c}$
a	**4750**	**0.8**	**1.21**
b	**3014**	**0.66**	**1.33**
c	**815**	**0.39**	**1.62**

2.2 The natural frequency index

A stationary uncracked rotor system is associated with well separated undamped natural frequencies ω_n^* (n=1,2,....). When the uncracked system is set to rotate, each of the ω_n^* is replaced by two frequencies: ω_{n+}^* which is slightly higher than ω_n^* and ω_{n-}^* which is slightly lower than ω_n^* [7, 26, 27]. The appearance of two frequencies, in place of one, for each mode is due to gyroscopic coupling moments which force the rotor system to vibrate with different frequencies in the vertical and horizontal planes. The presence of a crack will induce further changes from ω^*_{n+} to ω_{n+} and from ω^*_{n-} to ω_{n-}. Both ω_{n+} and ω_{n-} are associated with the same modal function.

The first index, used in this study, is based on the changes associated with the first natural frequencies as the crack develops. It is referred to as the "natural frequency index" and is denoted by ε_{f+} and ε_{f-} and given by:

$$\begin{aligned} \varepsilon_{f+} &= \left(\left| \omega_{1+} - \omega^*_{1+} \right| / \omega^*_{1+} \right) x\ 100 \\ \varepsilon_{f-} &= \left(\left| \omega_{1-} - \omega^*_{1-} \right| / \omega^*_{1-} \right) x\ 100 \end{aligned} \qquad \textbf{(3)}$$

where ω^*_{1+} and ω^*_{1-} represent the higher and lower natural frequencies of the first mode of the uncracked rotor and ω_{1+} and ω_{1-} represent the higher and lower natural frequencies of the first mode of the cracked rotor.

Extensive parametric studies were performed to investigate the variations of the natural frequency index ε_{f-} in terms of each parameter while maintaining the others constants. The studies were performed using the Finite Element model developed in [29] and backed up by experimental results [31]. Details of the experimental apparatus are given in [28]. A large number of cases were examined and the results carefully analysed.

2.2.1 Effect of crack depth and crack location

Typical plots are shown in figures (3, 4 and 5) for ε_{f-} versus the crack dimensionless position μ_C for configurations a, b and c . Figures 3, 4 and 5 are only plotted for greater speed ratio Λ_2. It can be noticed that for fixed values of the crack depth A_C, the frequency index reaches its maximal value when the crack is located in the neighbourhood of the position of maximal stress for all rotor systems under consideration. The frequency index increases as the crack depth increases. In fact, the index was found to be sensitive to variations in the crack depth which is a feature which could be exploited for practical identification of the depth of the crack.

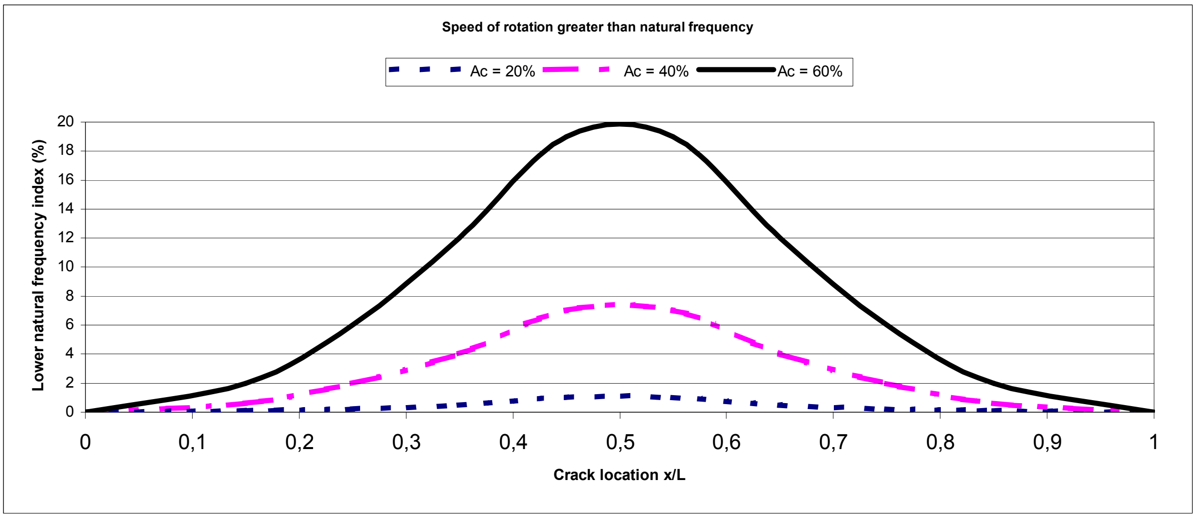

Figure 3: Natural frequency index vs crack location (cas A)

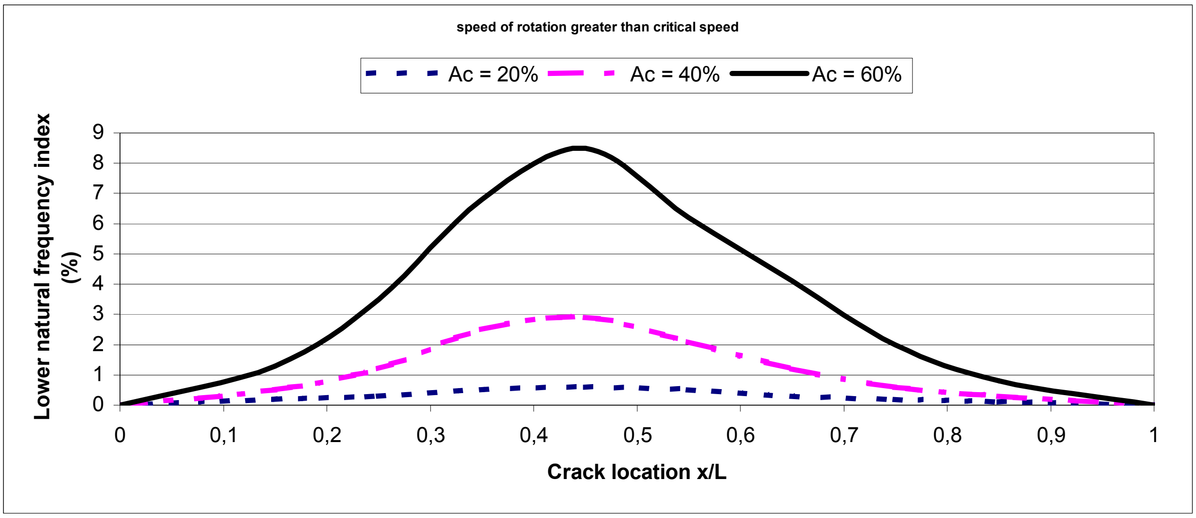

Figure 4: Natural frequency index vs crack location (cas B)

Unfortunately, it can be seen that the frequency index begins to be sensitive for crack depths greater than 40 % and only near to the maximal bending moment location. For crack depths lower than 30% and/or at crack locations far from the maximal stress location, the frequency index is lower than 1%, which is order of magnitude as the measurement error. The results show that the frequency index is less sensitive to the crack when a disk is present and for a cantilever rotor. Consequently the frequency analysis must be conducted with a very high precision in order to confirm the diagnosis [34].

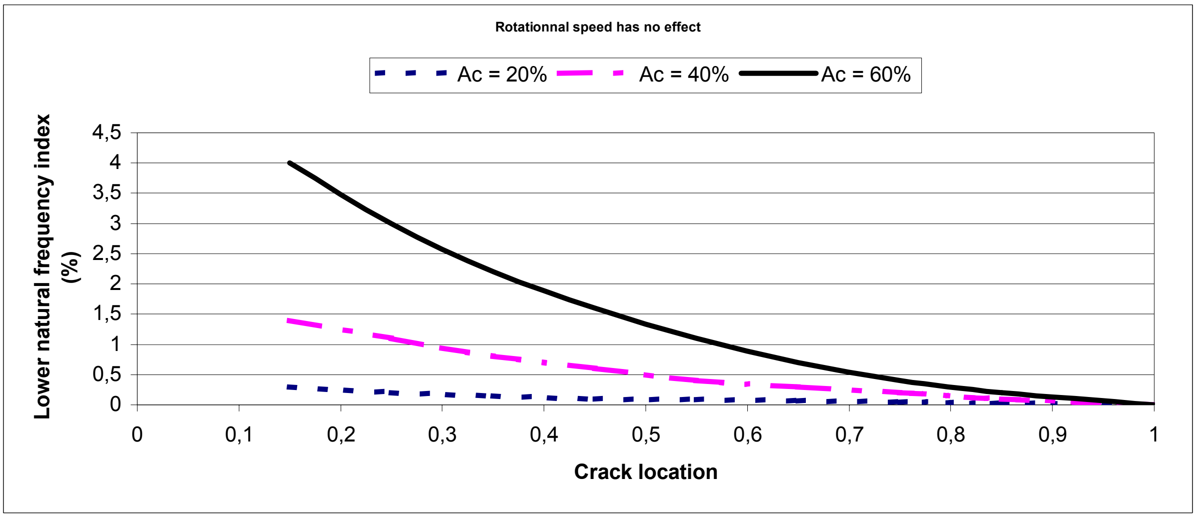

Figure 5: Natural frequency index vs crack location (cas C)

2.2.2 Effect of rotational speed

Figure (6) shows the effect of the speed of rotation on the frequency index for fixed values of crack location (35%) and depth (60%). This result is plotted for cas (a). It can be noticed that the index reaches its maximal value near the first critical speed of the rotor system. The same result has been obtained for all the configurations. It can be noticed that the rotational speed has a more significant effect in the range between 0.8 and 1.2 the critical speed.

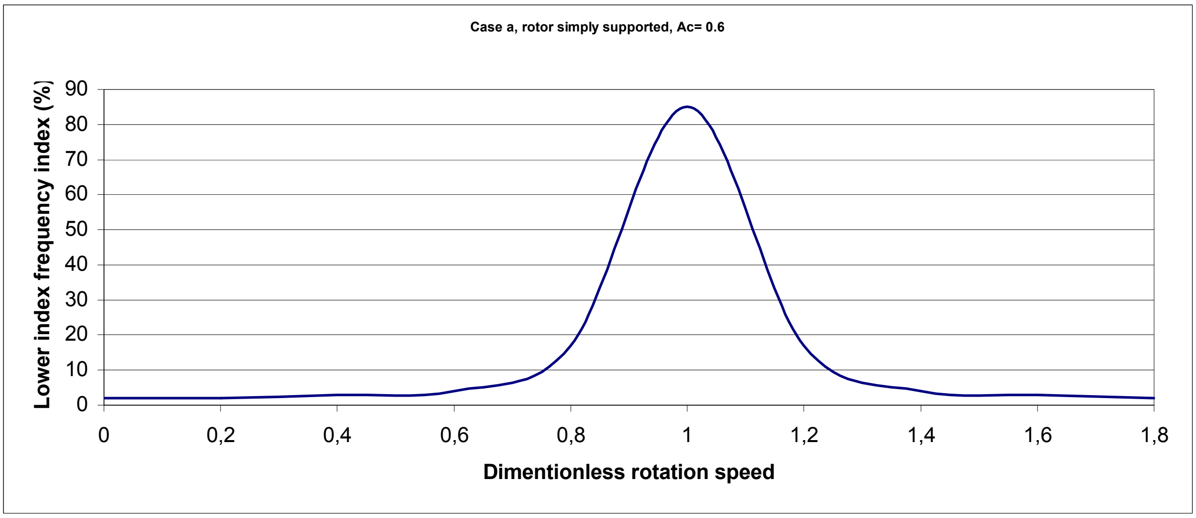

Figure 6 The natural frequency index vs the rotation speed (μ_c=35%)

2.2.3 Effect of rotor slenderness

A study of the effects of the slenderness parameter S on the frequency index is summarised in figure (7). Figure 7 has been plotted for a crack of 60% depth located at 35% of the rotor length. It also shows direct proportionality between ε_{f-} and S for all configurations. One can conclude that the frequency variation is more sensitive for stiffer shafts carrying a disk.

2.3 The line index

The second index is based on the changes associated with the mode shapes as a crack develops [35, 36, 37]. It was pointed out that the modal assurance criterion MAC [38, 39] was applied unsuccessfully to quantify the changes in the mode shapes. The MAC index was not sufficiently sensitive to the variation of the crack location and its depth. A new index is proposed which is referred to as the "line index" . The presence of a crack will induce further changes of the deflection of the first modal shape at given points X on the elastic line from w(X) to w*(X) where w(X) and $w^*(X)$ are the deflection of the first modal shape at a given location X on the shaft for the cracked and uncracked rotors, respectively.

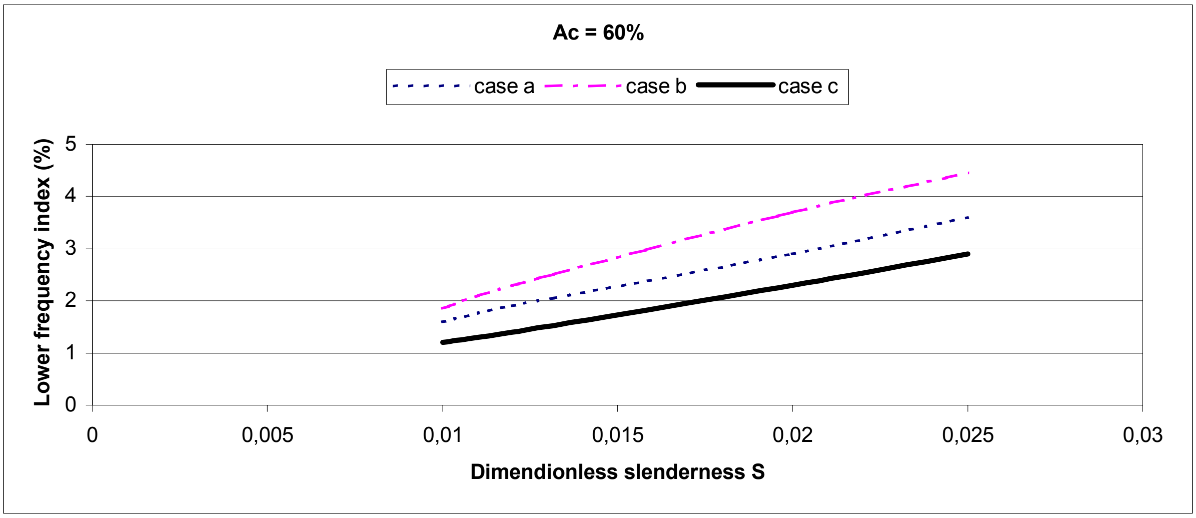

Figure 7 The natural frequency index vs the slenderness (μ_c=35%)

A dimensionless local deviation parameter can be defined at each location X on the shaft as follows:

$$LD(X) = \frac{\left|w(X) - w^*(X)\right|}{w^*(X)} \times 100 \qquad \textbf{(4)}$$

The elastic line index is defined as the maximum dimensionless local deviation parameter:

$$\varepsilon_e = \left|\frac{\Delta w(X_{mx})}{w^*(X_{mx})}\right| \times 100 = \left|\frac{w\,(X_{mx}) - w^*(X_{mx})}{w^*(X_{mx})}\right| \times 100 = LD(X_{mx}) \qquad \textbf{(5)}$$

where x_{max} is the location on the shaft of maximum deviation associated with $|\Delta w_{max}|$ and X_{mx} is the dimensionless location of maximum deviation = $\frac{x_{mx}}{L}$.

All mode shapes are normalised and are represented by vectors of unit length. The variables in equation (4) and (5) are the same as in figure (8) where $|\Delta w_{max}|$ is the maximal local deviation = $\left|w(X_{mx}) - w^*(X_{mx})\right|$.

Extensive computer simulations were performed using finite element model for all three configurations (a,b,c).

Figure 9 shows a modal amplitude applied to case (a) for A_C= 80%, S = 5% with a crack location at 15%. x is the location of a point on the shaft from left hand support and X is the dimensionless position of a point on the shaft = $\frac{x}{L}$.

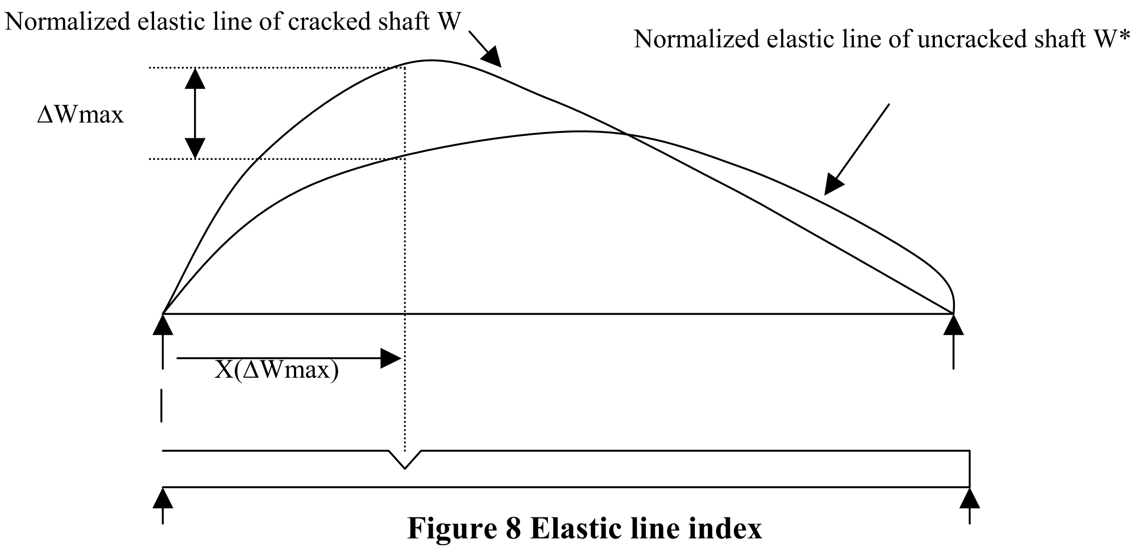

Figure 8 Elastic line index

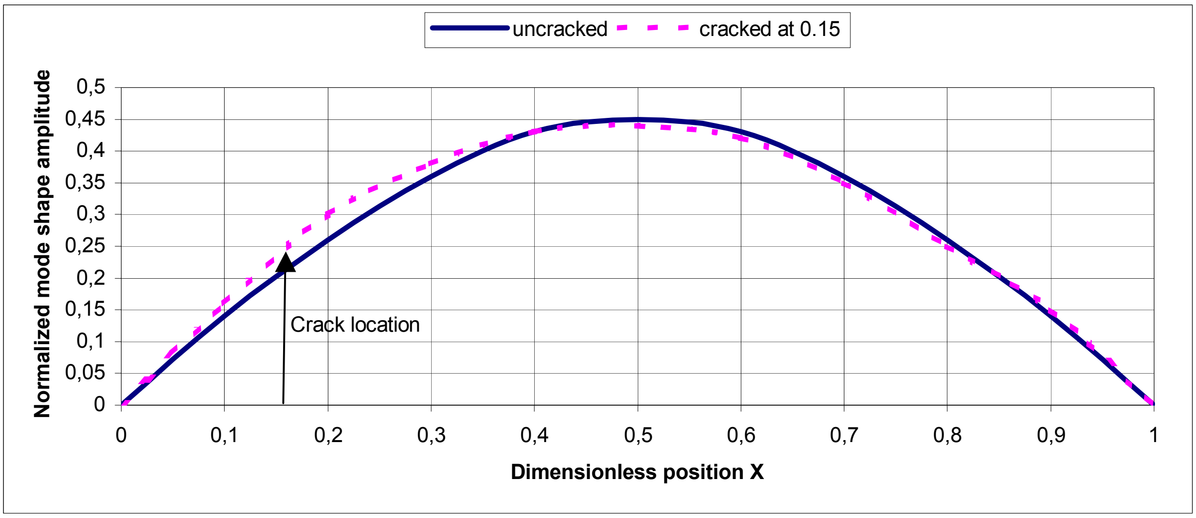

Figure 9 Mode shape of a cracked rotor

Figure 10 shows the local deviation index LD(X) computed from equation(4) and applied to the previous case (figure 10). The maximal of the line index appears close to the crack location. This analysis allows us to locate the crack when its location is unknown.

The same study has been applied bu using the operating deflection shape [40, 41] close to the natural frequency and similar results have been obtained. Consequently, the operating deflection shape can be used in place of the first mode shape. It has been shown that the elastic line index increases with the increase in the crack depth. The rate of the increase in the index depends, however, on the crack location. The line index seems more sensitive when cracks occur near locations and where the slope of the moment curve is maxima.

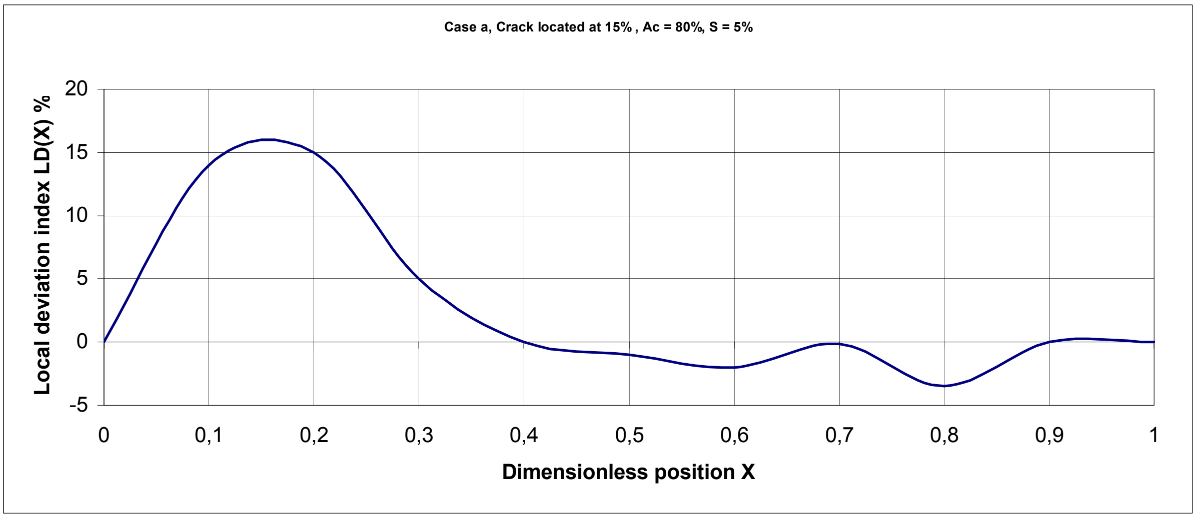

Figure 10 Line index

3. CONCLUSION

This paper presents parametric studies for open cracked rotor systems. In the case of open cracks, the roles played by the frequency index ε_{f-} and the line index ε_e and ε_{f-} are highlighted. The effects of crack depth, crack location, rotational speed and slenderness of the rotor on the natural frequency and the elastic line indexes are investigated in the case of three rotor configurations. The natural frequency index is found to be more sensitive to the crack depth, especially for rotating speeds ranging from 0.8 to 1.2 the critical speed. However the frequency index is less sensitive to low crack depths and does'nt allow an early detection of damage, unless a very high precision frequency analysis is conducted. The crack location can be predicted from the elastic line index. The newly introduced line index ε_e can be obtained by monitoring the operating deflection shape and allows us to locate the position of the crack μ_c.

4. REFERENCES

1. Chondros T.G., Dimarogonas A.D.,"Identification of cracks in welded joints of complex stuctures", Journal of sound and vibration, 69(4), 1980, pp 531-538.
2. Roitman N. and al.,"Identification of offshore platform structural damage using modal analysis techniques", Mechanical systems and signal processing, 6(3), 1992,pp 287-295.
3. Brinker R., Andersen P. and Cantieni R., Nov-Dec 2001, Identification and level 1 damage detecrtion of the Z24 highway bridge, Experimental techniques, Structural testing series, Part 13, pp 51-57
4. Law S.S. and al.,"Damage detection of a reinforced bridge deck using the frequency response function", IMAC 1992, pp 772-778.
5. Samman N.M., Biswas M.,"Employing pattern recognition for detecting cracks in bridge model", Int. journ. of analytical and experimental modal analysis, 6(1), 1991, pp 35-43.
6. Dimarogonas, A.D., 1996, Vibration of cracked structures: a state of the art review, Engineering fracture mechanics, 55 (5), pp 831-857.
7. Wauer J.,"On the dynamics of a cracked rotor, a literature survey", Applied mechanics review, 1990, pp 13-17.
8. Bently D.E. and Muszynska A., Detection of rotor cracks, Texas A&M 15th Turbomachinery Symposium Proceedings, Corpus Christi, Tx, 1986, pp 129-139.
9. Bently D.E. and Muszynska A., 1986, early detection of shaft cracks on fluid-handling machines, ASME, fluid engineering division, Vol. 46, pp 53-58.
10. Klein K, Guigné J.Y. and Swamidas A.S.J., 1994, monitoring change in modal parameters with fatigue, Proceeding of the 12th IMAC , pp 1792-1800.
11. Cawley P. and Adams R.D., The location of defects in structures from measurement of natural frequencies, Journal of strain analysis, Vol 14, No 2, 1979, pp 49-57.
12. Silva J.M.M. and Gomes A.J.M.A., 1994, crack identification of simple structural elements through the use of natural frequency variation, Proceeding of the 12th IMAC, pp 1728-1735.
13. Wyckaert K. and al.,"A feasability study for the application of frequency monitoring techniques in quality control", IMAC 1988, pp 921-927.
14. Brandon J.A. and Macleod D., 1990, An investigation into Interface mechanics in fracture, part 2: experimental results, 2nd international conference on interfaces in medecine and mechanics: interfaces 90, Bologna.
15. Yin Z.K. and al.,"Diagnosis of a slot fault on a frame structure", IMAC 1992, pp 549-553.
16. Chowdhury M.R., Ramirez M.,"A comparaison of the modal responses for defective versus non defective concrete test beams", IMAC 1992, pp 508-515
17. Gomes A.J.M., Montalvaoesilva J.M., "On the use of modal analysis for crack identification", IMAC 1992, pp 1108-1115.
18. Richardson M.H., Mannan M.A.,"Remote detection and location of structural faults using modal parameters", IMAC 1992, pp 502-507.
19. Srinivasan M.G., Kot C.A.,"Effect of damage on the modal parameters of a cylindrical shell", IMAC 1992, pp 529-535.

20. Yuen M.M.F., A numerical study of the eigenparameters of a damaged cantilever, Journal of sound and vibration, 103(3), 1985, pp 301-310.
21. Chondros T.G., 2000, Vibration of a beam with a breathing crack , Journal of Sound and Vibration, Vol 239.
22. Thomas M. and Lakis A.A., June 2002, Breathing crack detection by time-frequency analysis, Proceeding of the 30th Computer and industriel Engineering conference, Tinos grèce.
23. Frisswell M.I., Penny J.E.T.," A simple non linear model of a cracked beam", IMAC 1992, pp 516-521.
24. Leonard F, Lanteigne J, Lalonde S and Turcotte Y., 2000, Vibration behavior of a cracked beam, 18 th IMAC, San antonio, Texas, 7 p.
25. Safizadeh M.S., Lakis A.A., and Thomas M., 2000, Time-Frequency and their Application to Machinery Fault Detection, International Journal of Condition Monitoring and Diagnosis Engineering Management, 4(1), pp. 10-27.
26. Mayes W. and Davies W.G.,"The vibrational behaviour of a rotating shaft system containing a transverse crack", IMECHE, 1976,pp 53-64.
27. Waver J., 1990, Modelling and formulation of equations of motion for cracked rotating shafts, International Journal of Solid Structures, vol. 26, No. 8, pp 901-914.
28. Hamidi L., 1994, Développement d'une stratégie pour la détection de fissures et leur suivi dans les rotors tournant, PhD thesis, university of Sherbrooke, Quebec, Canada.
29. Hamidi L., Piaud J.B. and Massoud M., 1992, A study of cracks influence on the modal characteristics of rotors, 5th international conference on vibrations in rotating machinery, University of Bath U.K..
30. Hamidi L., Piaud J.B., Pastorel H., Mansour W.M. and Massoud M., 1994, Modal parameters for cracked rotors: modes and comparison, Journal of sound and vibration, 175 (2), pp 265-278,
31. Thomas M., Nguyen H., Hamidi L., Massoud M. and Piaud J.B., 1995, Detection of rotor cracks by experimental modal analysis, Transactions of the Canadian Society of Mechanical Engineering, Vol 19, No 2, pp 155-174.
32. Narkis Y., 1994, Journal of sound and vibration, 172(4), pp 549-558, 'Identification of crack location in vibrating simply supported beam'.
33. Dimarogonas A.D. and Papadopoulos C.A., 1983, Vibration of cracked shafts in bending, Journal of Sound and Vibration, Vol 91, No 4, pp 583-593.
34. Smail M., Thomas M. and Lakis A.A., Juin 1999, Utilisation de la modélisation ARMA pour la détection de fissures dans les rotors, Proceeding of the 3e Industrial Automation International conference (AIAI), ETS, Montreal, Qc, Canada, pp 21.1-21.4
35. Fox C.H.J.,"The location of defects in structures: a comparaison of the use of natural frequency and mode shape data", IMAC 1992, pp 522-528.
36. Mannan M.A.,"Detection and location of structural cracks using FRF measurements", IMAC 1990, pp 652-657.
37. Rizos P.F., Aspragathos N. and Dimarogonas A.D., 1990, Journal of sound and vibration, 138(3), pp 381-388,' Identification of crack location and magnitude in a cantilever beam from the vibration modes'.
38. Ewins D.J., "Modal testing: Theory and practice, J Wiley & Sons Inc, New York, 1984
39. Kim J.H. and al.,"Applications of the modal assurance criteria for detecting and locating structural faults", IMAC 1992, pp 536-540.
40. Dussing O., Staker C.H.,"Operational deflection shapes: background, measurements and applications", IMAC 1987, pp 1372-1378.

41. Xiaoqing Sun, Hardy H.R.,"A study on the operating shapes for non destructive evaluation in geotechnical engineering", IMAC 1992, pp 1237-1244.